Optimal Control of Wave Energy Converters: From Adaptive PI Control to Model Predictive Control

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Outline

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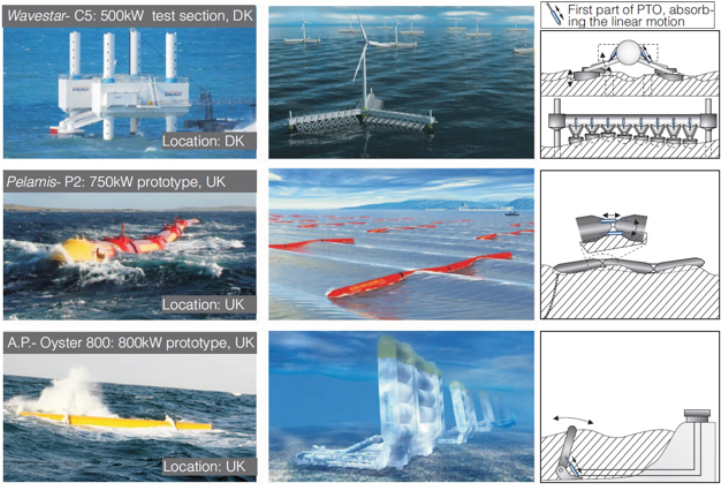
Wave Energy Converter

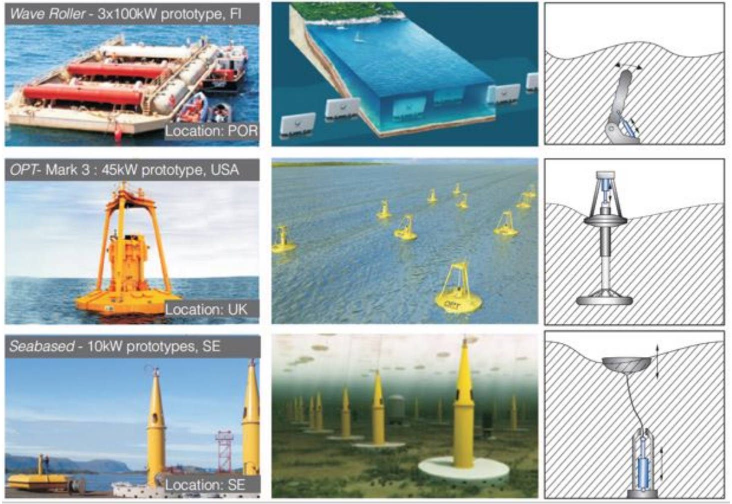
- WEC is a device which captures the power of waves and transforms it to electricity.
- The electricity generation from waves could amount to more than 2TW (18000TWh/year)

Wave energy distribution kW/m

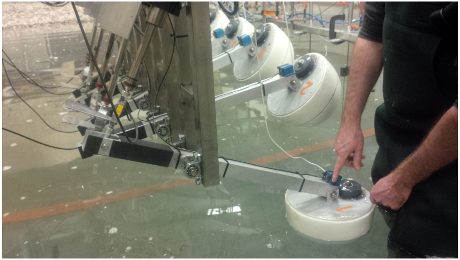
Some Prototypes

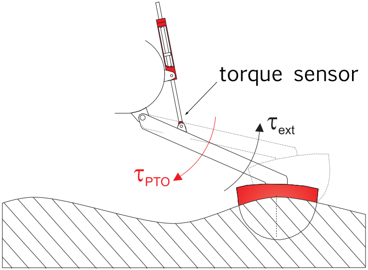


Some Prototypes, cont.



WEC modeling





- Newton's second law for rotation

$$J\ddot{\theta}(t) = M_{ex}(t) - M_{PTO}(t) - M_{hd}(t) - M_{rad}(t)$$

- $\theta(t)$: Float angle

WEC modeling, cont.

- J : Mass inertia moment
- $M_{ex}(t)$: Wave excitation moment
- $M_{PTO}(t)$: PTO moment
- $M_{hd}(t)$: Hydrostatic moment (due to gravity)

$$M_{hd}(t) = K\theta(t)$$

- $M_{rad}(t)$: Radiation moment (due to the float movement)

$$M_{rad}(t) = \int_{\tau=0}^{t} h(t-\tau)\dot{\theta}(\tau)d(\tau)$$

WEC modeling, cont.

- $J, K, h(t)$ can be derived from boundary element methods, estimated via dedicated experiments or both.
- State space equation,

$$\begin{cases} \dot{x}(t) = Ax(t) + B(M_{ex}(t) - M_{PTO}(t)) \\ y(t) = Cx(t) \end{cases}$$

- Where
  - $x(t)$: State
  - $y(t) = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix}$: Output

Control Objective

- Find $M_{PTO}(t)$ to maximize

$$P_{avr} = \frac{1}{T}\int_0^T \mu(t)\dot{\theta}(t)M_{PTO}(t)dt$$

- $\mu(t)$: efficiency coefficient, that depends on $\dot{\theta}(t)M_{PTO}(t)$

$$\mu = \begin{cases} 0.7, & \text{if } \dot{\theta}M_{PTO} \geq 0 \\ \frac{1}{0.7} = 1.43, & \text{if } \dot{\theta}M_{PTO} < 0 \end{cases}$$

- Pay two times more expensive to use energy from network

Control Objective, cont.

- Problems
  1. Even the control system is linear, the cost function is nonlinear.
  2. $x(t)$, $M_{ex}(t)$ are not directly available.
  3. Input and state constraints.
- Solutions
  1. Adaptive PI control.
  2. Model predictive control.

Outline

Intermittent adaptive PI control

- Current state of the art.
- Two phases: offline and online.
- Offline phase
  1. Select a set of representative sea states.
  2. Calculate the "optimal" PI gains for each sea state.
- Online phase
  1. Identify the current sea state, usually by its spectrum.
  2. PI gains are adapted online by a look up table.

Problems

- Offline phase
  - The PI gains are optimized by a griding method (brute-force search).
  - No proof of optimality.
- Online phase
  - Only average on-line estimations of sea states have been proposed, with time windows from 10 min. to 30 min.
  - Hence, the PI control gains are not continuously updated.

PI control for regular waves

- Assumption: $M_{ex}(t)$ is available and

$$M_{ex}(t) = A_w sin(wt + \phi)$$

- WEC model in the frequency domain

$$\frac{v(jw)}{M_{ex}(jw) - M_{PTO}(jw)} = \frac{1}{Z(jw)}$$

- Where
  - $v(jw) = \dot{\theta}(jw)$
  - $Z(jw)$: intrinsic impedance

PI control for regular waves, cont.

- Problem: Design a linear control law

$$M_{PTO}(jw) = K(jw)v(jw)$$

that maximizes the cost

- Denote

$$K(jw) = R_k + jX_k, \quad Z(jw) = R_z + jX_z$$

- Theorem

$$P_{avr} = \frac{A_w^2 \left(\mu R_k + \frac{1}{\pi}(\mu - \frac{1}{\mu})R_k(\frac{X_k}{R_k} - arctan(\frac{X_k}{R_k}))\right)}{2((X_z + X_k)^2 + (R_z + R_k)^2)}$$

Result of Falness et al., with $\mu = 1$

- With $\mu = 1$, recover the well-known result of Falness et al.

$$P_{avr} = \frac{A_w^2 R_k}{2((X_z + X_k)^2 + (R_z + R_k)^2)}$$

- P_{avr} is maximal, iff

$$X_k = -X_z, \; R_k = R_z$$

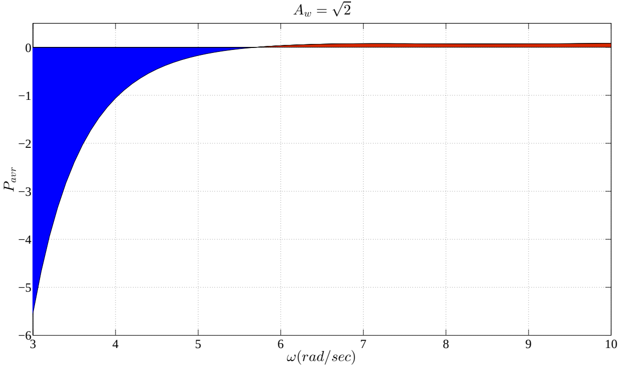
- Hence

$$\frac{v(jw)}{M_{ex}(jw)} = \frac{1}{2R_z(w)}$$

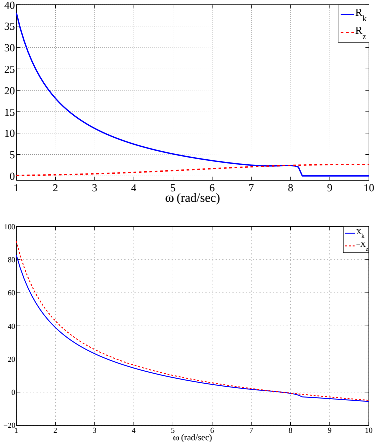
- P_{avr} is maximal iff $v(t)$ is in phase with $M_{ex}(t)$
- Are the results of Falness et al. correct also for $\mu < 1$?

Falness et al., cont.

- Let's take $X_k = -X_z$ and $R_k = R_z$ for P_{avr} with $\mu = 0.7$
- $P_{avr} < 0$ for all $\omega \leq 5.5(rad/sec)$ (where the wave has the most energy)
- Solution is not optimal, since one can take $R_k = X_k = 0$



PI control for regular waves, cont.



- It can be shown that the cost function is convex. Hence the optimal solution is unique.
- $v(t)$ is generally not in phase with $M_{ex}(t)$, since $X_k \neq -X_z$.

PI control for regular waves, cont.

- Up to now, for each regular wave $M_{ex}(t) = A_w sin(wt + \phi)$, the frequency response of the optimal controller is calculated

$$K(jw) = R_k(w) + jX_k(w)$$

- If $K(jw)$ is chosen as a PI controller

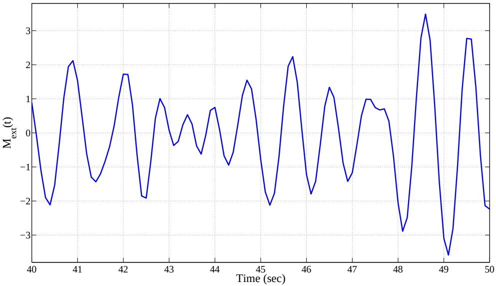
$$K(jw) = K_p + \frac{K_i}{jw}$$

- Then

$$K_p = K_k(w), \; K_i = -wX_k(w)$$

Problems

1. $M_{ex}(t)$ is not measurable.
2. Real $M_{ex}(t)$ is not a sinusoid.



3. Only the second problem is addressed now.

Frequency estimation

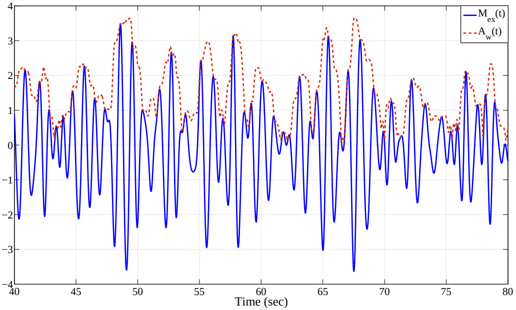
- Real $M_{ex}(t)$ is not a sinusoid, but it is not far from the sinusoid.
- Idea: approximate ON-LINE $M_{ex}(t)$ as

$$M_{ex}(t) = A_w(t) sin(w(t)t + \phi(t))$$

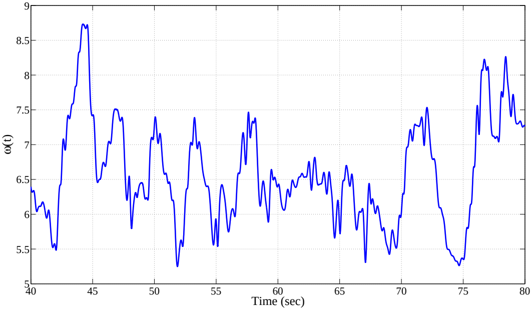
  where $A_w(t), w(t), \phi(t)$ are parameters.
- Classical problem.

Frequency estimation, cont.

- Unscented Kalman filter is used to estimate $A_w(t), w(t), \phi(t)$
- Details are not presented here.



Frequency estimation, cont.

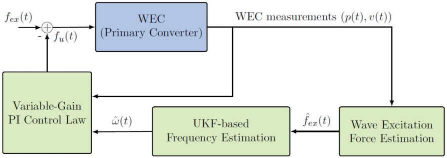


Adaptive PI control

The adaptive PI control algorithm is summarized as follows,

1. Measure $p(t)$, $v(t)$
2. Estimate $M_{ex}(t)$, $A_w(t), w(t), \phi(t)$

$$M_{ex}(t) = A_w(t)sin(w(t)t+\phi(t))$$

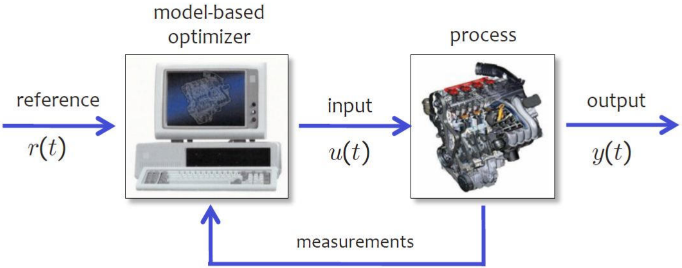


3. Calculate the control action

$$M_{PTO}(t) = K_p(w)v(t) + K_i(w)\int_0^t v(\tau)d\tau$$

Outline

Model predictive control - Basic concepts



A model of the process is used to predict the future evolution of the process to optimize the control signal

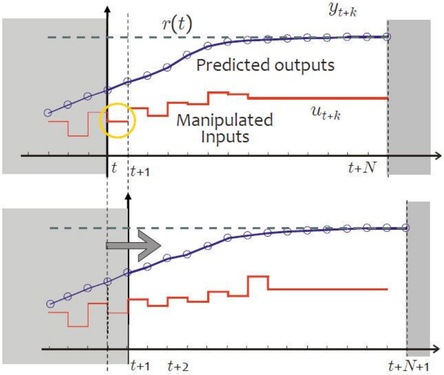
Receding horizon philosophy

At time k: Solve an optimal control problem over a finite future horizon of N steps

$$\min_{u(k),u(k+1),\dots,u(k+N-1)} J(k),$$

$$\text{s.t.} \begin{cases} u_{min} \leq u(k+j) \leq u_{max}, \\ y_{min} \leq y(k+j) \leq y_{max} \end{cases}$$

$J(k)$: cost function



- Only apply the first move $u^*(k)$
- At time $k+1$: Get new measurements, repeat the optimization. And so on ...

Why MPC for WEC?

- Maximize the extracted energy.
- Input and state constraints are incorporated in the design phase.
- Nonlinear efficiency coefficient is considered in the design phase.
- Wave prediction is explicitly used.

MPC problems

1. Wave excitation moment at the present and in the future are required
   1. Wave moment estimation
   2. Wave prediction
2. Nonlinear and non-convex optimization problem due to the nonlinear efficiency coefficient

Wave excitation moment estimation

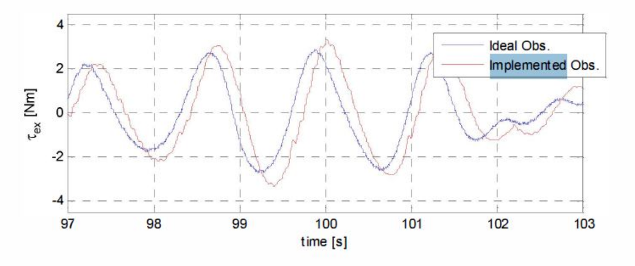
- Idea: Use a WEC model + measured outputs to estimate $M_{ex}(t)$
- Not a new idea
  - $M_{ex}(t)$ is decomposed as, P. Kracht et al., 2014

$$M_{ex}(t) = \sum_{j=1}^{m} \alpha_j(t) sin(\omega_j t) + \beta_j(t) cos(\omega_j t)$$

    where ω_j are chosen
  - $\alpha_j(t)$, $\beta_j(t)$ are estimated online using Luenberger observer

Wave excitation moment estimation, cont.

- The choice of ω_j is crucial
- The approach was experimentally tested at the Aalborg university
  - Slightly overestimate the amplitude
  - Non-negligible delay
- Not reliable in practice, since ω_j are time-varying



Random walk approach

- Idea: see $M_{ex}(k)$ as a state

$$M_{ex}(k+1) = M_{ex}(k) + \epsilon(k)$$

$\epsilon(k)$: variation of $M_{ex}(k)$, and is considered as a noise

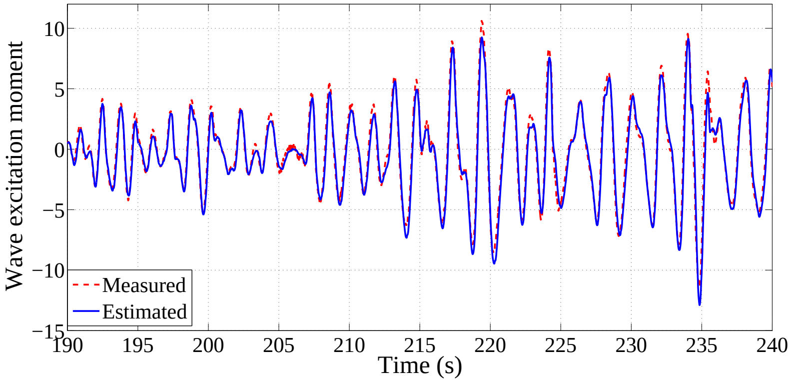
- Hence

$$\begin{cases} \begin{bmatrix} x \\ M_{ex} \end{bmatrix}^{+} & = \begin{bmatrix} A & B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ M_{ex} \end{bmatrix} - \begin{bmatrix} B \\ 0 \end{bmatrix} M_{PTO} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon \\ y & = [C \quad D] \begin{bmatrix} x \\ M_{ex} \end{bmatrix} - DM_{PTO} \end{cases}$$

Random walk approach, cont.

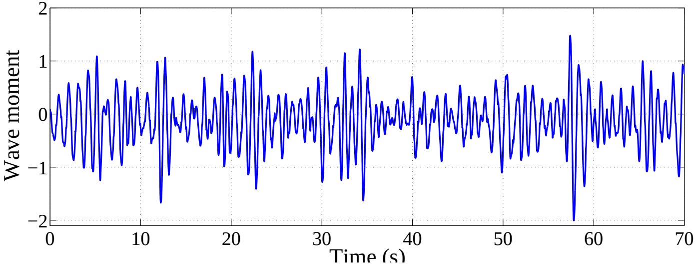
- The problem of estimating $M_{ex}(k)$ becomes the state estimation problem
- Kalman filter is used for this purpose
- Clearly, the approach can be used to estimate any kind of M_{ex} (not necessarily periodic)

Experimental results



Wave excitation moment prediction

- Given wave moments $y(k)$, $k = 0, 1, \ldots, k_0$ until time k_0
- Predict wave moments at times $k_0 + 1, k_0 + 2, \ldots, k_0 + N$



AR model based forecast

- Idea: Wave moment at time k is a linear function of a number p of its past values

$$y(k+1) = a_1 y(k) + a_2 y(k-1) + \ldots + a_p y(k-p+1)$$

- $\{a_1, a_2 \ldots, a_p\}$: parameters
- $\{a_1, a_2 \ldots, a_p\}$ can be found by minimizing the one step ahead prediction error

$$\min_{a_1, a_2, \ldots, a_p} \sum_{j=p+1}^{k} \left(y(j) - \sum_{i=1}^{p} a_i y(j-i) \right)^2$$

AR model based forecast, cont.

- Least square problem. Solution can be found analytically
- Result is not satisfactory for prediction
- Fusco's and Ringwood's idea: Long Rang Predictive Identification, i.e. minimizing not only the one step, but also the two-step, $\ldots$, the $h-$step prediction errors
- Nonlinear least square optimization problem. Batch-processing based solution.

Filter bank based forecast

- Previous method: iterative forecast
- Idea: forecast each horizon independently from the others
- N models for forecasting N steps ahead

$$\begin{cases} y(k+1) = a_{11}y(k) + a_{12}y(k-1) + \ldots + a_{1p}y(k-p+1) \\ y(k+2) = a_{21}y(k) + a_{22}y(k-1) + \ldots + a_{2p}y(k-p+1) \\ \vdots \\ y(k+N) = a_{N1}y(k) + a_{N2}y(k-1) + \ldots + a_{Np}y(k-p+1) \end{cases}$$

- Unknown parameters a_{ij}, $i = \overline{1, N}$, $j = \overline{1, p}$ are estimated by Kalman filter

Filter bank based forecast

- To forecast N steps ahead, one needs N Kalman filters
- Computational complexity is higher than iterative forecast
- Performance is better

Nonlinear MPC

- Finally, we are getting to the point
- Recall the state space equation of WEC

$$\begin{cases} x(k+1) = Ax(k) + BM_{ex}(k) - Bu(k) \\ y(k) = Cx(k) + DM_{ex}(k) - Du(k) \end{cases}$$

where $u(k) = M_{PTO}(k)$.

- Cost function at time k,

$$\max_{u(k),\ldots,u(k+N)} \sum_{j=0}^{j=N} \mu(k+j)v(k+j)u(k+j)$$

μ : nonlinear efficiency coefficient

Nonlinear efficiency function

- Taking into account directly μ in the cost gives rise to a nonlinear and nonconvex optimization problem

$$\begin{aligned} &\min_{U(k)} \left(U(k)^T H(\mu) U(k) + f(\mu)^T U(k) \right) \\ &\text{s.t. } u_{min} \leq u(k+j) \leq u_{max}, j = \overline{0, N} \end{aligned}$$

- Issues
  - Computational load
  - Difficult to investigate: feasibility, stability, robustness

Weighted MPC

- Consider again the cost function, for $\mu = 1$

$$J = \max_{U(k)} (u(k)v(k) + u(k)v(k+1) + u(k+1)v(k+1) + \dots)$$

- Weights are equal for all future costs
- This is not reasonable, since
  - Wave prediction performance is better for a short horizon, than for a large horizon.
  - It is better to put high weights on the current obtained energy for the first few time instants.

Weighted MPC, cont.

- In the result

$$J = \max_{U(k)} (w_0 u(k) v(k) + w_0 u(k) v(k+1) + w_1 u(k+1) v(k+1) + \ldots)$$

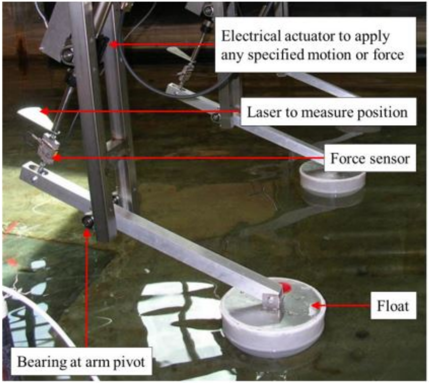
- $w_0, w_1, \ldots, w_{N-1}$: tunning coefficients
- We usually choose

$$w_0 \geq w_1 \geq \ldots \geq w_{N-1}$$

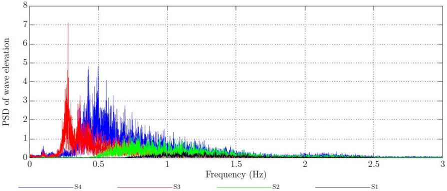
- QP problem

Outline

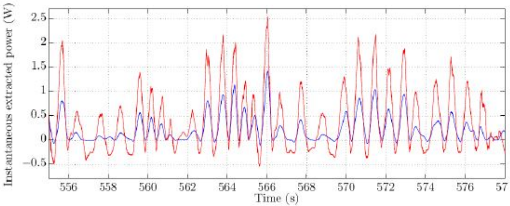
Experimental Setup

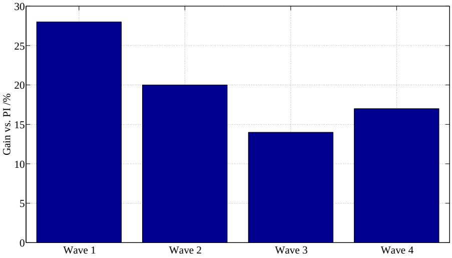


- Tests in Aalborg University basin on a pivoting-buoy point absorber.
- 4 different sea states are considered.



Experimental results, cont





Conclusions

- Two main solutions are proposed for WECs with non-perfect PTO
  - Adaptive PI: optimal control for regular waves, wave force and dominant wave frequency estimation.
  - Model predictive control: wave estimation, wave prediction, weighted MPC, QP problem.
- Successfully implemented for a real system.
- Perspective: Decentralized/distributed control, stochastic MPC.

THANK YOU