

A small step towards a systematic design of effective and implementable NMPC setting

The Python-package MPC_tuner



```
class MPC_tuner:
    def __init__(self, target):
        self.target = target
    def set_target(self, target):
        self.target = target
    def set_options(self, options):
        self.options = options
    def call(self):
        mpc.call(self.target, self.options)
```

Mazen Alamir

CNRS / University of Grenoble Alpes / Gipsa-lab



Recalls on Nonlinear Model Predictive Control (NMPC)

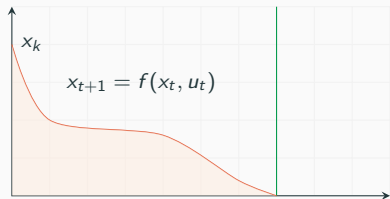
Real-Time considerations

Design of NMPC setting

The MPC_tuner package

Conclusion and future extensions

Recalls on Nonlinear Model Predictive Control (NMPC)

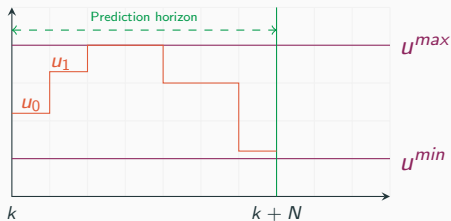
Predicted state profile $x(\cdot)$


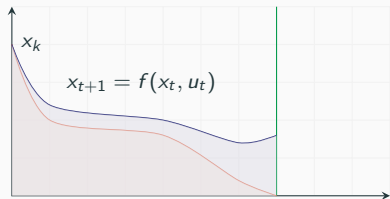
\forall candidate control sequence

$$\mathbf{u}_k := (u_0, \dots, u_{N-1})$$

a cost can be predicted

$$J(\mathbf{u}_k | x_k)$$

 Candidate control profile \mathbf{u}_k


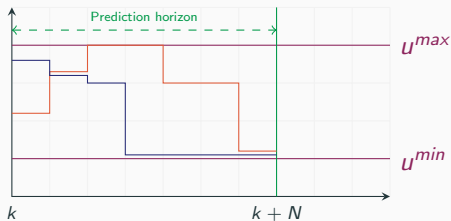
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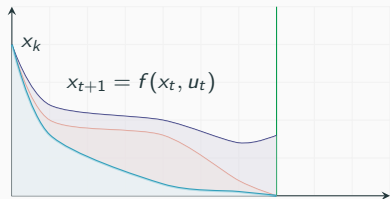
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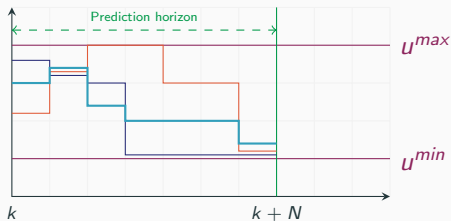
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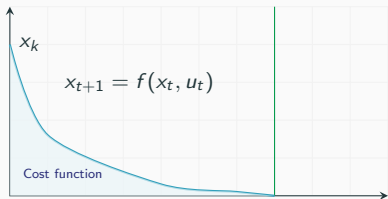
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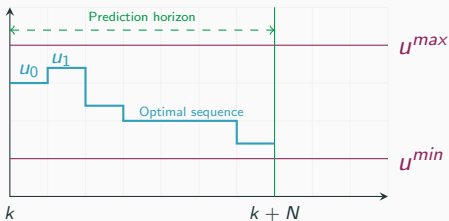
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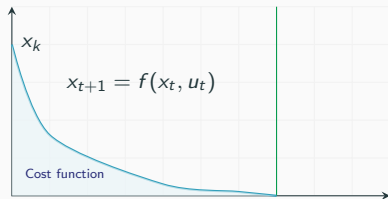
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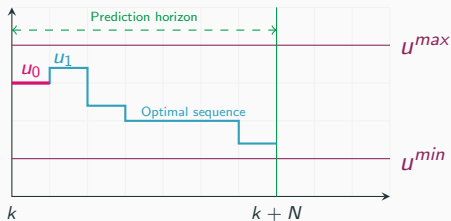
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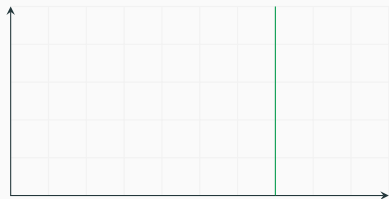
 Candidate control profile \mathbf{u}_k


MPC feedback

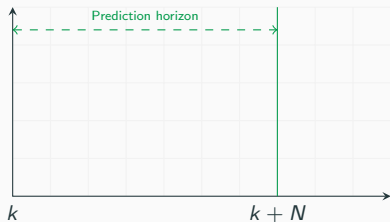
$$u_k = u_0^*(x_k) \quad \text{where}$$

$$J(\mathbf{u}^* | x_k) = \min_{\mathbf{u} \in \mathcal{U}(x)} J(\mathbf{u} | x_k)$$

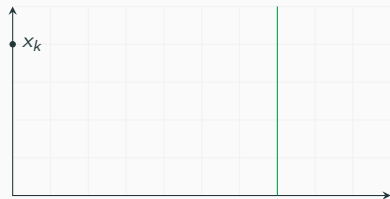
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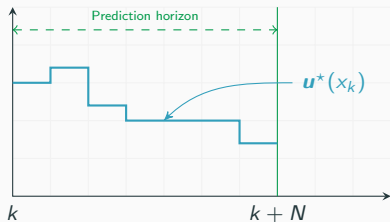
Control profile $u(\cdot)$



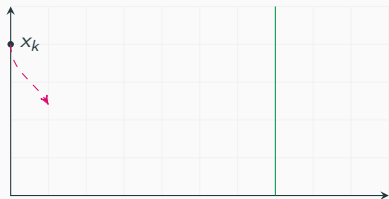
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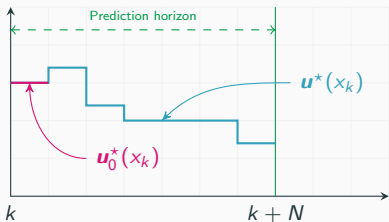
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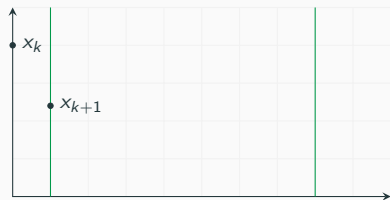
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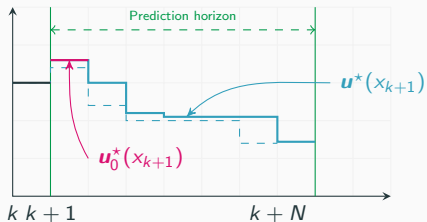
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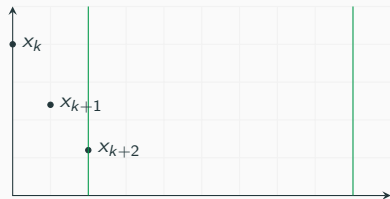


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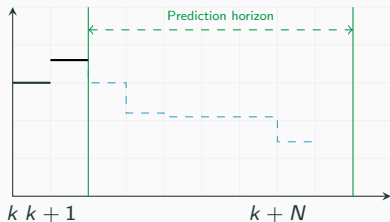


NMPC ← The **Receding-horizon** principle in action

Predicted state profile $x(\cdot)$

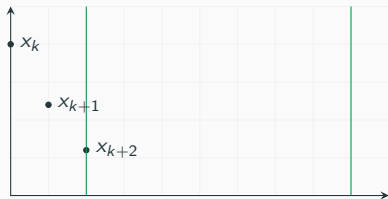


Control profile $u(\cdot)$



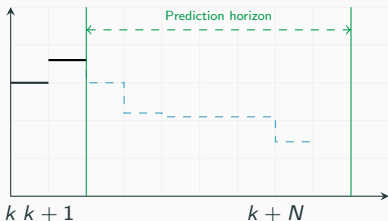
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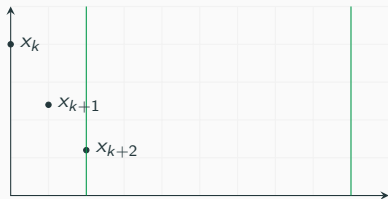
- ✓ At each updating instant
- ✓ The prediction horizon is shifted
- ✓ **A new optimization problem is defined**
- ✓ and **solved**
- ✓ The first action is applied
- ✓ until the next updating instant
- ✓ The process is repeated
- ✓ ...

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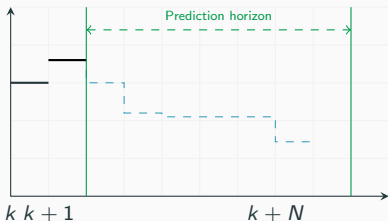


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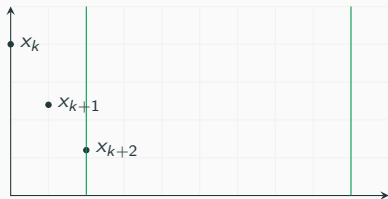
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--- REAL-TIME AGNOSTIC DEFINITION ---

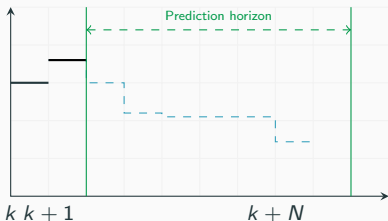
No reference to computation time!

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--- REAL-TIME AGNOSTIC DEFINITION ---

No reference to computation time!

We'll come back to this later...
let's first examine the **optimization problem!**

MPC feedback

$$u_k = u_0^*(x_k) \quad \text{where}$$

$$J(u^* | x_k) = \min_{u \in \mathbb{U}(x)} J(u | x_k)$$

More precisely ...



Automatica

Volume 36, Issue 6, June 2000, Pages 789-814



Survey Paper

Constrained model predictive control: Stability and optimality ☆

D.Q. Mayne^a, J.B. Rawlings^b, C.V. Rao^b, P.O.M. Scokaert^c

$$\min_{\mathbf{u}} \left[\Psi(x_{k+N}) + \sum_{i=1}^N \ell(x_{k+i}, u_{k+i-1}) \right]$$

$$\text{under } \begin{cases} x_{k+N} \in \mathbb{X}_f \\ x_{k+i} \in \mathbb{X} \\ u_{k+i-1} \in \mathbb{U} \end{cases}$$

† where Ψ is a control-Lyapunov-function inside \mathbb{X}_f .



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- ✗ Never used outside academic toy examples!

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Don't even try!

Automatica 75 (2017) 288–292



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journal homepage: www.elsevier.com/locate/automatica

Brief paper

Contraction-based nonlinear model predictive control formulation without stability-related terminal constraints^a

Mazen Alamir

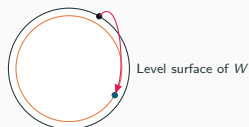
CNRS/Gipsu-lab, University of Grenoble Alpes, France

$$\min_{(\mathbf{u}, q)} \left[\alpha \min_{i=1}^q W(x_{k+i}) + z \sum_{i=1}^q \ell(x_{k+i}, u_{k+i-1}) \right]$$

$$\text{under } \left\{ \begin{array}{l} (\mathbf{u}, q) \in \mathbb{U}^q \times \{1, \dots, N\} \\ x_{k+i} \in \mathbb{X} \end{array} \right.$$

where z is an internal state of the controller

- ✓ **Contractive formulations**
- ✓ **Prediction horizon is a decision variable.**



Accept transient increase of the cost function

→ multi-step Lyapunov function

Automatica 87 (2018) 455–459



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Technical communique

Stability proof for nonlinear MPC design using monotonically increasing weighting profiles without terminal constraints*

Mazen Alamir

CNRS, Univ. Grenoble Alpes, Gipsa-lab, F-38000, Grenoble, France

- ✓ Enhances stability with short prediction horizons
- a smoother version of the terminal state constraints

$$\min_{\mathbf{u}} \left[\sum_{i=1}^N [i/N]^m \ell(x_{k+i}, u_{k+i-1}) \right]$$

$$\text{under } \left| \begin{array}{l} \mathbf{u} \in \mathbb{U}^N \\ x_{k+l} \in \mathbb{X} \end{array} \right.$$

where $m \in \mathbb{N}$

Automatica 125 (2021) 109420



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journal homepage: www.elsevier.com/locate/automatica

Brief paper

A new formulation of Economic Model Predictive Control without terminal constraint[☆]

Mazen Alami^{a,*}, Gabriele Pannocchia^b

^a Univ. Grenoble Alpes, CNRS, Grenoble INP, GIPSA-lab, 38000 Grenoble, France

^b Department of Civil and Industrial Engineering, University of Pisa, 56126 Pisa, Italy

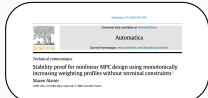
- ✓ Economic MPC
- ✓ Standard proof of stability

$$\min_{\mathbf{u}} \gamma \left[\ell_{k+N} + \rho_d \Delta_{k+N} \right] + \sum_{i=1}^N \left[\ell_{k+i} + \rho_d \Delta_{k+i} \right]$$

$$\text{under } \begin{cases} \mathbf{u} \in \mathbb{U}^N \\ x_{k+l} \in \mathbb{X} \end{cases}$$

$$\text{where } \Delta_k := \|x_k - x_{k-1}\|$$

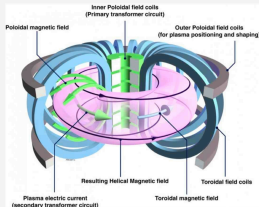
Does the formulation matter?



QUESTION: Does the formulation really matter?

The **impact** of formulations!

Cryogenic Refrigerators - Problem statement



Source: <https://www.euro-fusion.org>

Why?

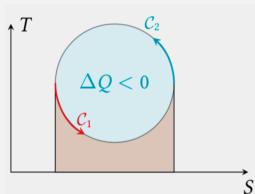
Provide **refrigeration capacity** to cool down the supra-conducting coils used to accelerate the plasma in Nuclear Fusion Reactors (ITER, JT60)



F. Bonne (CEA)



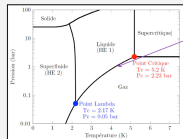
P. Bonny (CEA)



How?

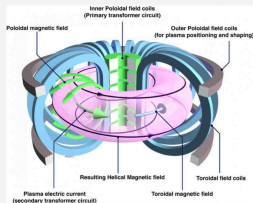
Force a thermodynamic **fluid** to make a **counter-clock cycle** in the (S, T)=(Entropy, Temperature) plan.

$$\int dQ = \underbrace{\int_{c_1} TdS}_{>0} + \underbrace{\int_{c_2} TdS}_{<<0}$$



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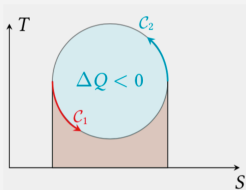
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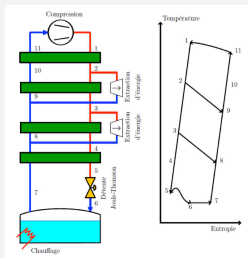
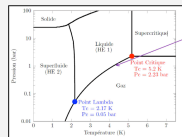
P. Bonny (CEA)



How?

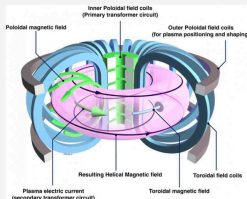
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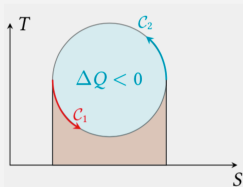
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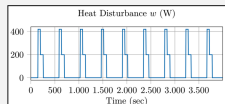
P. Bonnay (CEA)



How?

Force a thermodynamic fluid to make a counter-clock cycle in the (S, T) =(Entropy, Temperature) plan.

$$\int dQ = \underbrace{\int_{C_1} TdS}_{>0} + \underbrace{\int_{C_2} TdS}_{<<0}$$



Model

$$\begin{aligned}x^+ &= Ax + Bu + Gw \\ y &= Cx + Du\end{aligned}$$

- $(x, u, w) \in \mathbb{R}^{24} \times \mathbb{R}^3 \times \mathbb{R}$
- w unpredictable heat pulse
- $u \in \mathbb{U}$ two valves+heating (bath)

Objective

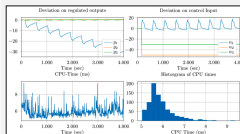
Keep the station sustainable despite of unmeasured and unpredictable w .

The **impact** of formulations!

Cryogenic refrigerator: The impact of formulation

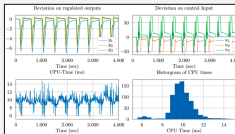
Cost function

$$\sum_{i=1}^N \left[\|y_{k+i}\|_{Q_y}^2 + \epsilon \|x_{k+i}\|^2 \right]$$



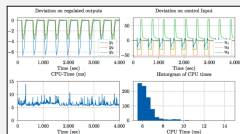
Cost function

$$\sum_{i=1}^N \left[\|y_{k+i}\|_{Q_y}^2 + \epsilon \|x_{k+i}\|^2 \right] + \epsilon_f \|x_{i+N}\|^2$$



Cost function

$$\sum_{i=1}^N \left[\left(\frac{i}{N}\right)^m \left[\|y_{k+i}\|_{Q_y}^2 + \epsilon \|x_{k+i}\|^2 \right] \right]$$



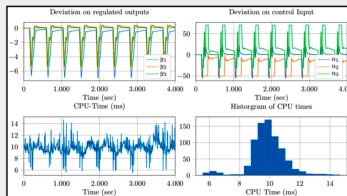
Optimization problems are solved using the **GUROBI-Python-3** framework implemented on a Mac-PowerBook 2.9GHz (High Sierra OS version 10.13.3).

The **impact** of formulations!

Cryogenic refrigerator: The impact of formulation

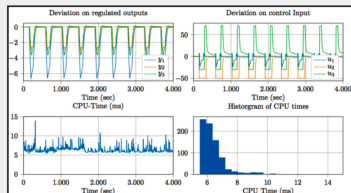
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Cost function

$$\sum_{i=1}^N \left[\frac{i}{N} \right]^m \left[\|y_{k+i}\|_{Q_y}^2 + \epsilon \|x_{k+i}\|^2 \right]$$



Does the formulation matter?



QUESTION: Does the formulation really matter?

ANSWER: Of course!

A single parameterized formulation

Standard $\left[\Psi(x_{k+N}) + \sum_{i=1}^N \ell_{k+i} \right]$ s.t. $c_j(x_{k+i}, u_{k+i-1}) \leq 0$

Economic $\gamma \left[\ell_{k+N} + \rho_d \Delta_{k+N} \right] + \sum_{i=1}^N \left[\ell_{k+i} + \rho_d \Delta_{k+i} \right]$ s.t. $c_j(x_{k+i}, u_{k+i-1}) \leq 0$

Variable weight $\left[\sum_{i=1}^N [i/N]^m \ell(x_{k+i}, u_{k+i-1}) \right]$ s.t. $c_j(x_{k+i}, u_{k+i-1}) \leq 0$

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Unified structure parameterized by $\rho_f, \rho_d, m, \rho_{cstr}, N$

$$\min_{\mathbf{u}} \left[\rho_f \Psi(x_{k+N}, \rho_d \Delta_{k+N}) + \sum_{i=1}^N [i/N]^m \left[\ell_{k+i} + \rho_d \Delta_{k+i} \right] + \rho_{cstr} \max_{j=1}^{n_c} [c_j(x_{k+i}, u_{k+i-1})]_+ \right]$$

A single parameterized formulation

How to choose the parameters of the formulation?

- ✓ **Constraints** must be respected over the closed-loop trajectory
- ✓ Closed-loop **stability** should be guaranteed
- ✓ The feedback must be **real-time** implementable

Unified structure parameterized by $\rho_f, \rho_d, m, \rho_{cstr}, N$

$$\min_{\mathbf{u}} \left[\rho_f \Psi(x_{k+N}, \rho_d \Delta_{k+N}) + \sum_{i=1}^N [i/N]^m [\ell_{k+i} + \rho_d \Delta_{k+i}] + \rho_{cstr} \max_{j=1}^{n_c} [c_j(x_{k+i}, u_{k+i-1})]_+ \right]$$

A single **parameterized formulation**

How to choose the parameters of the formulation?

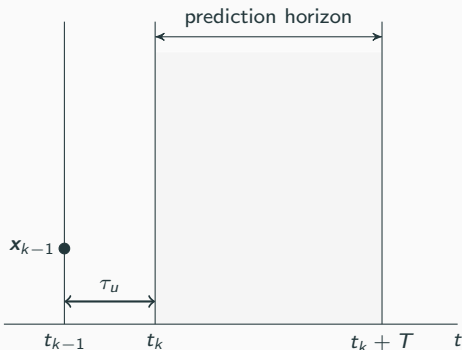
- ✓ **Constraints** must be respected over the closed-loop trajectory
- ✓ Closed-loop **stability** should be guaranteed
- ✓ The feedback must be **real-time** implementable ← SEE NEXT

Unified structure parameterized by $\rho_f, \rho_d, m, \rho_{cstr}, N$

$$\min_{\mathbf{u}} \left[\rho_f \Psi(x_{k+N}, \rho_d \Delta_{k+N}) + \sum_{i=1}^N [i/N]^m [\ell_{k+i} + \rho_d \Delta_{k+i}] + \rho_{cstr} \max_{j=1}^{n_c} [c_j(x_{k+i}, u_{k+i-1})]_+ \right]$$

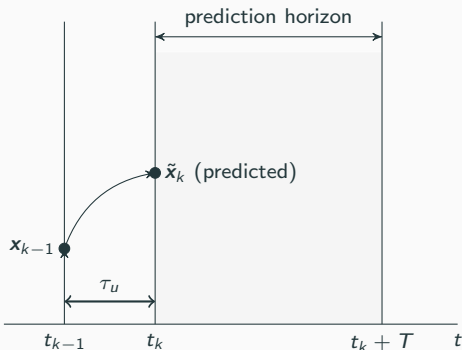
Real-Time considerations

MPC: The Control **Updating Period** τ_u



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1. Predict $\tilde{\mathbf{x}}_k$ (CPU-negligible)
2. During $[t_{k-1}, t_k]$ Compute $\mathbf{u}^*(\tilde{\mathbf{x}}_k)$



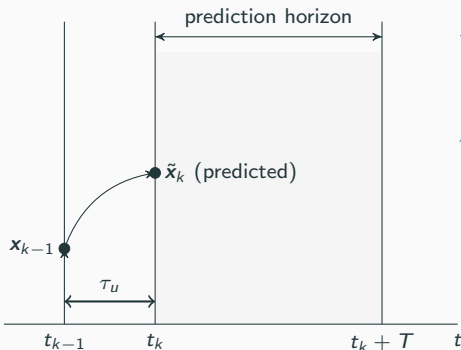
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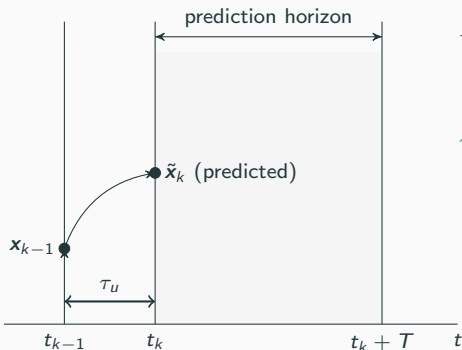
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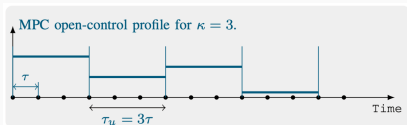
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τ_u is a highly impacting implementation setting **choice**.



Implementation options



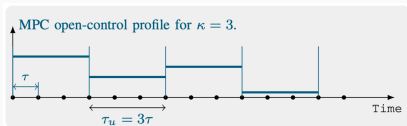
τ_u The updating period.

τ Basic sampling period[†]

[†] Largest τ making RK(τ) appropriate

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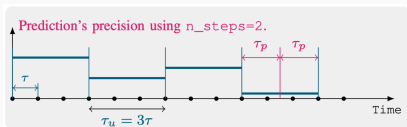


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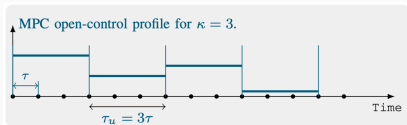
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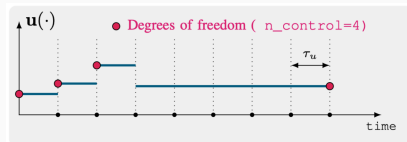


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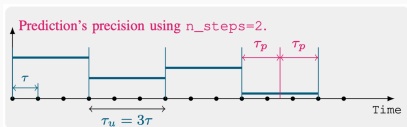
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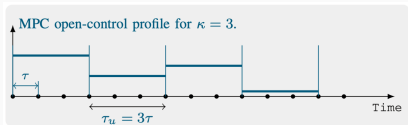
$$n_{\text{control}} \times n_u \quad \text{d.o.f.}$$



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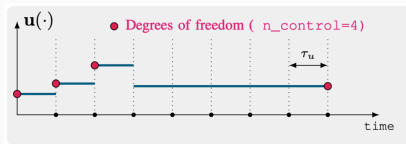


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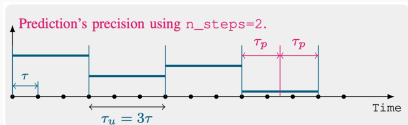


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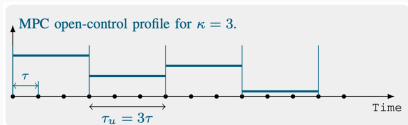
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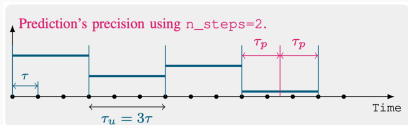


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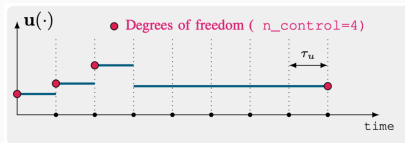
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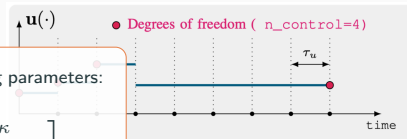
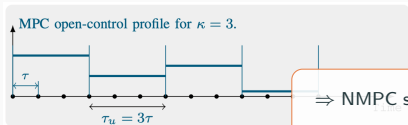
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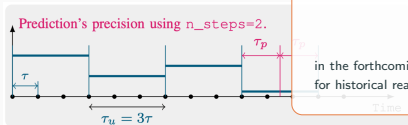
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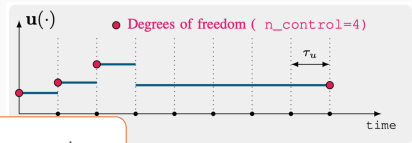
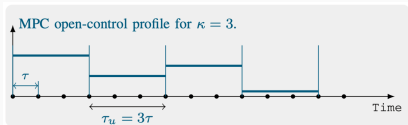
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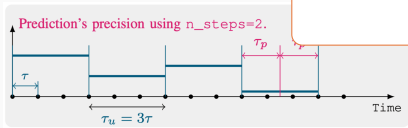
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Design of NMPC setting

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Given the following items:

- ✓ Problem ingredients: (dynamics, constraints, $\overbrace{\text{parameterized cost}}^{\psi, \ell}$)
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What does this precisely mean?

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# candidates	$\eta = 0.1$	$\eta = 0.05$	$\eta = 0.01$	$\eta = 0.001$
1	132	264	1317	13164
5	154	308	1536	15354
10	163	326	1628	16280
100	193	386	1930	19299
1000	223	445	2225	22249

Table 1: The required $\text{card}(\mathcal{A})$ as a function of the precision parameter η for a confidence parameter of $\delta = 10^{-3}$ and a number of admitted failure = 1.

The values of n_{samples} needed to be **confident at $(1-\delta)100\%$** that the properties are satisfied with a probability greater than **$(1-\eta)100\%$**

[Alamo et al. Randomized strategies for probabilistic solution of uncertain feasibility and optimization problems, IEEE TAC, 2009]

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 - ✓ Π is rather high dimensional non convex set!
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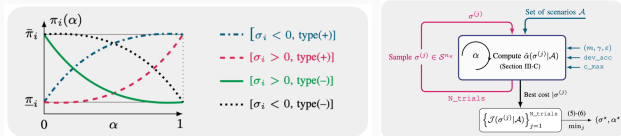
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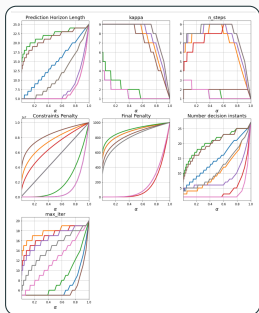
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[M. Alamir. A Framework and a python-package for Real-time NMPC parameters settings. <http://arxiv.org/abs/2309.17238>. 2023.]

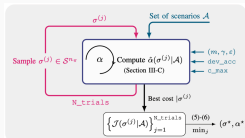
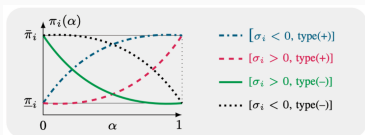
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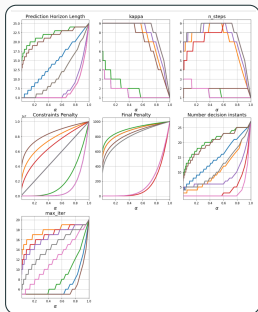
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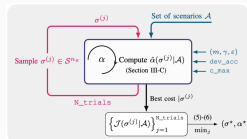
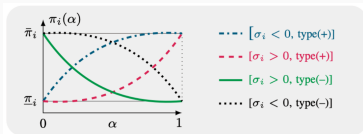
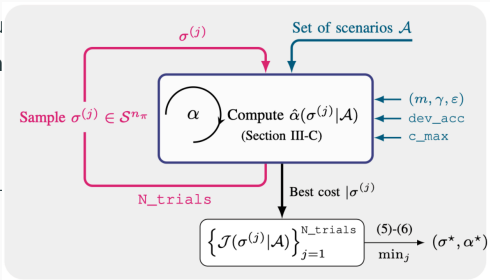
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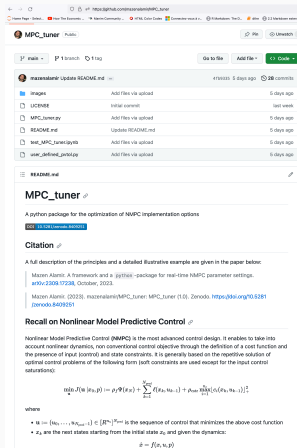


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The MPC_tuner package

The MPC_tuner package: An overview

https://github.com/mazenalamir/MPC_tuner



The screenshot shows the GitHub repository for MPC_tuner. The repository name is 'MPC_tuner' and it is public. The commit history shows several recent updates, including README files and source code files. The README file is selected, showing the following content:

MPC_tuner

A python package for the optimization of NMPC implementation options

[View Source](#)

Citation

A full description of the principles and a detailed illustrative example are given in the paper below:

- Mazen Alamir, A framework and a `@@@`-package for real-time NMPC parameter settings, [arXiv:2209.07238](#), October, 2023.
- Mazen Alamir (2023), mazenalamir/MPC_tuner: MPC_tuner (1.0), Zenodo, <https://doi.org/10.5281/zenodo.8409251>

Recall on Nonlinear Model Predictive Control

Nonlinear Model Predictive Control (NMPC) is the most advanced control design. It enables to take into account nonlinear dynamics, non conventional control objective through the definition of a cost function and the presence of input (control) and state constraints. It is generally based on the repetitive solution of optimal control problems of the following form (soft constraints are used except for the input control saturations):

$$\min_{\mathbf{u}} J(\mathbf{u} | \mathbf{x}_0, \mathbf{p}) := \mathbf{p}_T^T \mathbf{f}(\mathbf{x}_T) + \sum_{k=0}^{T-1} \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \lambda_{\text{sum}} \sum_{k=0}^{T-1} [\mathbf{c}(\mathbf{x}_k, \mathbf{u}_k)]^2$$

where

- $\mathbf{u} := [\mathbf{u}_0, \dots, \mathbf{u}_{T-1}] \in [\mathbb{R}^m]^{T \times m}$ is the sequence of control that minimizes the above cost function
- \mathbf{x}_k are the next states starting from the initial state \mathbf{x}_0 and given the dynamics:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p})$$

The MPC_tuner package: An overview

MPC_tuner Public

main 1 branch 1 tag

Go to file Add file Code

mazenalamir · Update README.md · 4181055 · 5 days ago · 28 commits

- Images Add files via upload 5 days ago
- LICENSE Initial commit last week
- MPC_tuner.py Add files via upload 5 days ago
- README.md Update README.md 5 days ago
- test_MPC_tuner.py Add files via upload 5 days ago
- user_defined_pb.py Add files via upload 5 days ago

README.md

MPC_tuner

A python package for the optimization of NMPC implementation options

<https://doi.org/10.48550/arXiv.2309.17238>

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Recall on Nonlinear Model Predictive Control (NMPC)

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$$\min_{\mathbf{u}} J(\mathbf{u} | \mathbf{x}_0, \mathbf{p}) := \int_0^N \ell(\mathbf{x}, \mathbf{u}, \mathbf{p}) dt + \sum_{k=1}^N \ell(\mathbf{x}_k, \mathbf{u}_k) + \lambda_{\text{hor}} \sum_{k=1}^N \|\mathbf{x}(\mathbf{x}_k, \mathbf{u}_k) - \mathbf{x}^*\|^2$$

where

- $\mathbf{u} := [\mathbf{u}_0, \dots, \mathbf{u}_{N-1}] \in [\mathbb{R}^m]^{N \times m}$ is the sequence of control that minimizes the above cost function
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 $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p})$

https://github.com/mazenalamir/MPC_tuner

MPC_tuner package / user-defined pb

MAIN CLASSES

Sigma

```
__init__(nGrid)
show_df(nGrid)
plot_phi(nGrid)
```

MPC

```
__init__(pb, sigma, alpha)
feedback(x, p, q, z0)
sim_cl(sc, z0, optim_par)
```

MAIN FUNCTIONS

```
Generate_A(pb, nb, nsb)
Design_MPC(pb, S, A, optim_par)
```

pb=Container()

ATTRIBUTES

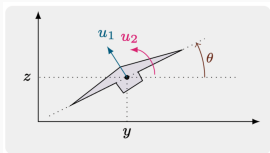
```
nx, np, nq, tau, nc
u_min, u_max
x_min, x_max
q_min, q_max
p_nom, p_std
```

METHODS

```
ode(x, u, p, q)
constraints(x, u, p, q)
stage_cost(x, u, p, q)
final_penalty(x, u, p, q)
generate_cloud(n)
```

[M. Alamir. A Framework and a python-package for Real-time NMPC parameters settings. <http://arxiv.org/abs/2309.17238>. 2023.]

The MPC_tunner package: A use-case

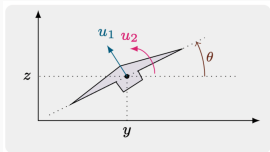


$$\ddot{y} = -u_1 \sin \theta + p_1 u_2 \cos \theta$$

$$\ddot{z} = u_1 \cos \theta + p_1 u_2 \sin \theta - 1$$

$$\ddot{\theta} = p_2 u_2$$

The MPC_tunner package: A use-case



$$\ddot{y} = -u_1 \sin \theta + p_1 u_2 \cos \theta$$

$$\ddot{z} = u_1 \cos \theta + p_1 u_2 \sin \theta - 1$$

$$\ddot{\theta} = p_2 u_2$$

$$x := (y, z, \theta, \dot{y}, \dot{z}, \dot{\theta})$$

$$x_d := (q_1, q_2, 0, \dots, 0)$$

$$\ell := \|x - x_d\|_Q^2 + \|u - u_d\|_R^2$$

$$\Psi := \rho_f \|x - x_d\|_Q$$

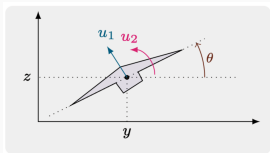
$$\gamma := 0.98,$$

$$u \in [-50, +50]^2$$

$$|\dot{\theta}| \leq 1 \text{ and } |\theta| \leq \pi$$

$$\tau = 0.02$$

The MPC_tunner package: A use-case



Design parameter	min-value	max-value
N_pred	5	25
κ	1	10
ρ_f	1	10^3
rho_cstr	10^3	10^7
max_iter	5	20

Definition of Π boundaries.

$$\ddot{y} = -u_1 \sin \theta + p_1 u_2 \cos \theta$$

$$\ddot{z} = u_1 \cos \theta + p_1 u_2 \sin \theta - 1$$

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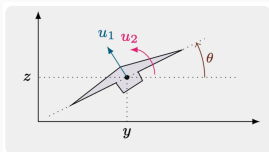
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The MPC_tunner package: A use-case



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$$x := (y, z, \theta, \dot{y}, \dot{z}, \dot{\theta})$$

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N_pred	5	25
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ρ_f	1	10^3
rho_cstr	10^3	10^7
max_iter	5	20

Definition of Π boundaries.

```
import numpy as np
from casadi import vertcat

class Container:
    def __init__(self):
        pass

pvtol = Container()
pvtol.nx, pvtol.nu = 6, 2
pvtol.n_p, pvtol.n_q = 2, 4
pvtol.u_min = np.array([-5e1, -5e1])
pvtol.u_max = np.array([+5e1, +5e1])
pvtol.x_min = np.array([-2, -2, -0.8 * np.pi, \
                        -0.1, -0.1, -0.1])
pvtol.x_max = -pvtol.x_min
pvtol.q_min = [-1, -1, 1, np.pi]
pvtol.q_max = [+1, +1, 1, np.pi]
pvtol.p_nom = pvtol.p
pvtol.p_std, pvtol.tau, pvtol.nc = 0.1, 0.02, 4
```

The user-defined file (part 1)

```
#-----
def p_vtol(x, u, p):
    xdot = vertcat(...)
    return xdot
#-----
def constraints(x, u, p, q):
    d_theta_dt_max, theta_max = q[2], q[3]
    c = vertcat(x[5]/d_theta_dt_max-1,...)
    return c
#-----
def stage_cost(x, u, p, q):
    ...
    return ell
#-----
def final_penalty(x, p, q):
    ...
    ef = 0
    for i in range(6):
        ef += q_xf[i] * (x[i] - xd[i]) ** 2
    return ef
#-----
def generate_cloud(nSamples=None):
    x_min, x_max = pvtol.x_min, pvtol.x_max
    ...
    A_sc = Container()
    A_sc.x0, A_sc.p, A_sc.q = X, P, Q
    return A_sc
#-----
pvtol.ode = p_vtol
pvtol.constraints = constraints
pvtol.final_penalty = final_penalty
pvtol.stage_cost = stage_cost
pvtol.generate_cloud = generate_cloud
```

The user-defined file (part 2)

The MPC_tunner package: A use-case

```
from user_defined_pvtol import pvtol
from pyMPC import Design_MPC, Sigma, generate_A, OptimPar

# Generate the set of scenarios
nb, nsb = 30, 10
A = generate_A(pvtol, nb, nsb)

# Generate the set of candidate sigma's
N_trials = 100
S = [Sigma() for _ in range(N_trials)]

# Set the Design meta-parameters and run the Design
optim_par = OptimPar(gam=0.98, c_max=0.1,
                    dev_acc=1.0, T=0.5)

R_design_log = Design_MPC(pvtol, S, A, optim_par)
```

$$\gamma := 0.98,$$

$$u \in [-50, +50]^2$$

$$|\dot{\theta}| \leq 1 \text{ and } |\theta| \leq \pi$$

$$\tau = 0.02$$

```
pvtol.q_min = [-1, -1, 1, np.pi]
pvtol.q_max = [+1, +1, 1, np.pi]
pvtol.p_nom = pvtol.p
pvtol.p_std, pvtol.tau, pvtol.nc = 0.1, 0.02, 4
```

The user-defined file (part 1)

```
#-----
def p_vtol(x, u, p):
    xdot = vertcat(...)
    return xdot
#-----
def constraints(x, u, p, q):
    d_theta_dt_max, theta_max = q[2], q[3]
    c = vertcat(x[5]/d_theta_dt_max-1,...)
    return c
#-----
def stage_cost(x, u, p, q):
    ...
    return ell
#-----
def final_penalty(x, p, q):
    ...
    ef = 0
    for i in range(6):
        ef += q_xf[i] * (x[i] - xd[i]) ** 2
    return ef
#-----
def generate_cloud(nSamples=None):
    x_min, x_max = pvtol.x_min, pvtol.x_max
    ...
    A_sc = Container()
    A_sc.x0, A_sc.p, A_sc.q = X, P, Q
    return A_sc
#-----
pvtol.ode = p_vtol
pvtol.constraints = constraints
pvtol.final_penalty = final_penalty
pvtol.stage_cost = stage_cost
pvtol.generate_cloud = generate_cloud
```

The user-defined file (part 2)

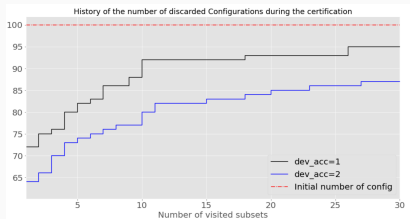
Results: Admissible design settings

dev_acc = 1

	kappa	N_pred	n_steps	n_ctr	rho_cstr	rho_final	tau	tau_u	max_iter	cost	alpha
0	9	7	3	8	9.646822e+06	178.800537	0.02	0.18	11	7859.487689	0.750
1	4	6	2	4	9.102911e+06	24.259781	0.02	0.08	16	4404.438416	0.625
2	2	5	2	2	7.937212e+06	1.243896	0.02	0.04	8	2437.847089	0.250
3	8	7	5	2	1.250875e+06	2.951172	0.02	0.16	18	3401.306977	0.500
4	9	6	7	7	9.170123e+06	8.804688	0.02	0.18	6	3755.852208	0.500

dev_acc = 2

	kappa	N_pred	n_steps	n_ctr	rho_cstr	rho_final	tau	tau_u	max_iter	cost	alpha
0	7	17	4	12	8.409123e+06	16.609375	0.02	0.14	5	3210.056159	0.250
1	9	7	3	8	9.646822e+06	178.800537	0.02	0.18	11	7859.487689	0.750
2	9	20	2	19	8.409123e+06	820.515021	0.02	0.18	8	3766.654430	0.250
3	6	17	3	16	6.124112e+06	3.778133	0.02	0.12	5	3284.618189	0.375
4	4	6	2	4	9.102911e+06	24.259781	0.02	0.08	16	4404.438416	0.625
5	9	5	8	4	6.259375e+05	2.951172	0.02	0.18	18	3126.158301	0.500
6	8	12	2	11	7.519303e+04	1.054939	0.02	0.16	17	2891.563097	0.375
7	7	9	5	2	2.442162e+06	15.537363	0.02	0.14	19	3583.879784	0.625
8	9	5	8	6	9.350676e+06	38.215650	0.02	0.18	18	4611.172897	0.625
9	2	12	2	2	8.846257e+06	1.146503	0.02	0.04	7	3408.564392	0.375
10	6	6	6	2	5.282910e+05	1.146503	0.02	0.12	18	3355.846137	0.375
11	9	18	2	13	3.536180e+06	125.875000	0.02	0.18	12	3250.814430	0.125
12	6	19	2	2	7.430228e+06	2.951172	0.02	0.12	14	3360.845813	0.125



Results of two different runs of the **Design_MPC** for two targeted device **dev_acc=1** and **dev_acc=2**.

Both use **100** sampled candidate design settings.

$$\text{card}(\mathcal{A}) = 270 > N_{\text{certification}}(\eta = 0.05, \delta = 10^{-3}) = 264$$

Results: Admissible design settings

dev_acc = 1

	kappa	N_pred	n_steps	n_ctr	rho_cstr	rho_final	tau	tau_u	max_iter	cost	alpha
0	9	7	3	8	9.646822e+06	178.800537	0.02	0.18	11	7859.487689	0.750
1	4	6	2	4	9.102911e+06	24.259781	0.02	0.08	16	4404.438416	0.625
2	2	5	2	2	7.937212e+06	1.243896	0.02	0.04	8	2437.847089	0.250
3	8	7	5	2	1.250875e+06	2.951172	0.02	0.16	18	3401.306977	0.500
4	9	6	7	7	9.170123e+06	8.804688	0.02	0.18	6	3755.852208	0.500

dev_acc = 2

	kappa	N_pred	n_steps	n_ctr	rho_cstr	rho_final	tau	tau_u	max_iter	cost	alpha
0	7	17	4	12	8.409123e+06	16.609375	0.02	0.14	5	3210.056159	0.250
1	9	7	3	8	9.646822e+06	178.800537	0.02	0.18	11	7859.487689	0.750
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Design parameter	min-value	max-value
N_pred	5	25
κ	1	10
ρ_f	1	10^3
rho_cstr	10^3	10^7
max_iter	5	20

Small portion of configurations is eligible

5%
13%

dev_acc=1
dev_acc=2

Results of two different runs of the Design_MPC for two targeted device dev_acc=1 and dev_acc=2.

Both use 100 sampled candidate design settings.

$$\text{card}(\mathcal{A}) = 270 > N_{\text{certification}}(\eta = 0.05, \delta = 10^{-3}) = 264$$

Results: Admissible design settings

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ρ_f	1	10^3
rho_cstr	10^3	10^7
max_iter	5	20

High values of ρ_f lead to unfeasibility

Recall that random sampling of

$$\rho_f \in [1, 1000]$$

is applied but only rather small values appear in the resulting admissible settings.

Results of two different runs of the Design_MPC for two targeted device dev_acc=1 and dev_acc=2.

Both use 100 sampled candidate design settings.

$$\text{card}(\mathcal{A}) = 270 > N_{\text{certification}}(\eta = 0.05, \delta = 10^{-3}) = 264$$

Results: Admissible design settings

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dev_acc = 2

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6	8	12	2	11	7.515030e+04	1.054939	0.02	0.16	17	2891.563097	0.375
7	7	9	5	2	2.442162e+06	15.537363	0.02	0.14	19	3583.879784	0.625
8	9	5	8	6	9.350676e+06	38.215650	0.02	0.18	18	4611.172897	0.625
9	2	12	2	2	8.846257e+06	1.146503	0.02	0.04	7	3408.564392	0.375
10	6	6	6	2	5.282910e+05	1.146503	0.02	0.12	18	3355.846137	0.375
11	9	18	2	13	3.536180e+06	125.875000	0.02	0.18	12	3250.814430	0.125
12	6	19	2	2	7.430228e+06	2.951172	0.02	0.12	14	3360.845813	0.125

Results of two different runs of the Design_MPC for two targeted device dev_acc=1 and dev_acc=2.

Both use 100 sampled candidate design settings.

$$\text{card}(\mathcal{A}) = 270 > N_{\text{certification}}(\eta = 0.05, \delta = 10^{-3}) = 264$$

Design parameter	min-value	max-value
N_pred	5	25
κ	1	10
ρ_f	1	10^3
rho_cstr	10^3	10^7
max_iter	5	20

High values of ρ_f lead to unfeasibility

Recall that random sampling of

$$\rho_f \in [1, 1000]$$

is applied but only rather small values appear in the resulting admissible settings.

High values of ρ_{cstr} are necessary

Only high values of ρ_{cstr} appear in the admissible settings

Results: Admissible design settings

dev_acc = 1

	kappa	N_pred	n_steps	n_ctr	rho_cstr	rho_final	tau	tau_u	max_iter	cost	alpha
0	9	7	3	8	9.646822e+06	178.800537	0.02	0.18	11	7859.487889	0.750
1	4	6	2	4	9.102911e+06	24.259781	0.02	0.08	16	4404.438416	0.625
2	2	5	2	2	7.937212e+06	1.243896	0.02	0.04	8	2437.847089	0.250
3	8	7	5	2	1.250875e+06	2.951172	0.02	0.16	18	3401.306977	0.500
4	9	6	7	7	9.170123e+06	8.804688	0.02	0.18	6	3755.852208	0.500

dev_acc = 2

	kappa	N_pred	n_steps	n_ctr	rho_cstr	rho_final	tau	tau_u	max_iter	cost	alpha
0	7	17	4	12	8.409123e+06	16.609375	0.02	0.14	5	3210.056159	0.250
1	9	7	3	8	9.646822e+06	178.800537	0.02	0.18	11	7859.487889	0.750
2	9	20	2	19	8.409123e+06	820.515021	0.02	0.18	8	3766.654430	0.250
3	6	17	3	16	6.124112e+06	3.778133	0.02	0.12	5	3284.618189	0.375
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⇒ The scenarios are constraints-challenging

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Large eligible possibilities of prediction horizons

$$N_{\text{pred}} \in [0.2, 1.26] \quad \text{dev_acc}=1$$
$$N_{\text{pred}} \in [0.48, 3.24] \quad \text{dev_acc}=2$$

Conclusion and future extensions

The MPC_tuner package: Conclusion and future extensions

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Recall on Nonlinear Model Predictive Control

Nonlinear Model Predictive Control (NMPC) is the most advanced control design. It enables to take into account nonlinear dynamics, non conventional control objective through the definition of a cost function and the presence of input (control) and state constraints. It is generally based on the repetitive solution of optimal control problems of the following form (soft constraints are used except for the input control saturation):

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where

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[DOI: 10.26434/chemrxiv-2023-01011](https://doi.org/10.26434/chemrxiv-2023-01011)

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$$J(u) = \int_0^{T_f} (x^T Q x + u^T R u) dt + \lambda \|x(T_f) - x_f\|^2$$

where

- $u := [u_0, \dots, u_{T-1}] \in \mathbb{R}^{m \times T}$ is the sequence of control that minimizes the above cost function
- x_0 are the next states starting from the initial state x_0 and given the dynamics:

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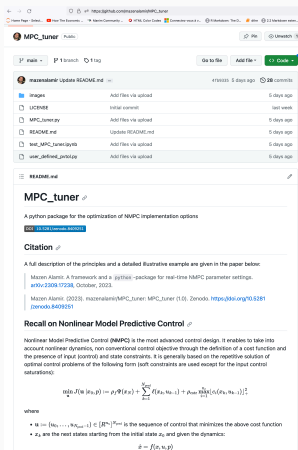
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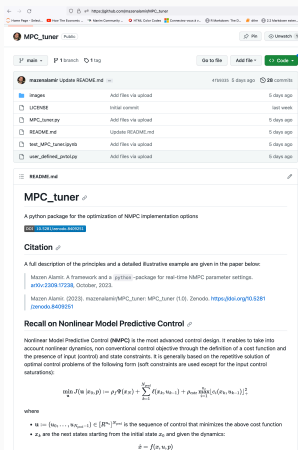
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$$\min_{\mathbf{u}} J(\mathbf{u}) = J(\mathbf{x}_0, \mathbf{y}) := \rho_f \Phi(\mathbf{x}_N) + \sum_{k=0}^{N-1} \left(\rho_{\text{cost}} \mathbb{1}_{\mathbb{R}^+}[\alpha(\mathbf{x}_k, \mathbf{u}_k, \mathbf{y}_k)] \right)^2$$

where

- $\mathbf{u} := [\mathbf{u}_0, \dots, \mathbf{u}_{N-1}] \in \mathbb{R}^{n \times N}$ is the sequence of control that minimizes the above cost function
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The MPC_tuner package: Conclusion and future extensions

https://github.com/mazenalamir/MPC_tuner

MPC_tuner · Public

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- images · Add files via upload · 5 days ago
- LICENSE · Initial commit · last week
- MPC_tuner.py · Add files via upload · 5 days ago
- README.md · Update README.md · 5 days ago
- test_MPC_tuner.py · Add files via upload · 5 days ago
- user_defined_penalty · Add files via upload · 5 days ago

MPC_tuner

A python package for the optimization of NMPC implementation options

[View on PyPI](#) [View on GitHub](#)

Citation

A full description of the principles and a detailed illustrative example are given in the paper below:

- Mazen Alamir, A framework and a jupyter package for real-time NMPC parameter settings, arXiv:2309.17238, October, 2023.
- Mazen Alamir (2023), mazenalamir/MPC_tuner: MPC_tuner (1.0), Zenodo, <https://doi.org/10.5281/zenodo.8409251>

Recall on Nonlinear Model Predictive Control

Nonlinear Model Predictive Control (NMPC) is the most advanced control design. It enables to take into account nonlinear dynamics, non conventional control objective through the definition of a cost function and the presence of input (control) and state constraints. It is generally based on the repetitive solution of optimal control problems of the following form (soft constraints are used except for the input control saturation):

$$\min_{\mathbf{u}} J(\mathbf{u}) = \int_0^N L(\mathbf{x}_k, \mathbf{u}_k) + \sum_{k=0}^{N-1} \lambda_k \|\mathbf{x}_k - \mathbf{x}_k^{\text{ref}}\| + \lambda_{\text{term}} \|\mathbf{x}_N - \mathbf{x}_N^{\text{ref}}\|$$

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$$\min_{\mathbf{u}} J(\mathbf{u}) = \int_{t_0}^{t_f} L(\mathbf{x}_k, \mathbf{u}_k) + \sum_{k=0}^{N-1} f(\mathbf{x}_k, \mathbf{u}_{k+1}) + \rho_{\text{term}} \mathbb{1}_{\{\|\mathbf{x}(t_f, \mathbf{u}_0, \mathbf{u}_k)\| \geq \epsilon\}}$$

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$$\min_{\mathbf{u}} J(\mathbf{u}) = \gamma \mathcal{P}(\mathbf{x}_0) + \sum_{k=0}^{N-1} \ell(\mathbf{x}_k, \mathbf{u}_k) + \rho_{\text{term}} \min_{\mathbf{z}} \|\mathbf{z}\|_2^2$$

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Each example needed \approx 1hr under MacBook Pro, 2.4GHz intel Corei9

Thank you!