Distributed Moving Horizon Estimation with pre-estimation using Extended Kalman Filter for Nonlinear Measurements

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Overview

Context and applications

Distributed State Estimation (DSE)

- Each agent/node (limited sensing, computation and communication capabilities):
  1. Sharing information obtained by its embedded sensors within its neighborhood
  2. Computing a local state estimate by fusing information (consensus)

- Providing increased autonomy, scalability, computational efficiency, fault-tolerance, etc.
- Applications: multi-vehicle localization, surveillance or tracking missions by sensor networks.

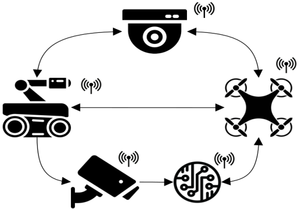

Fig. 1: Communication graph representing the network between nodes/agents

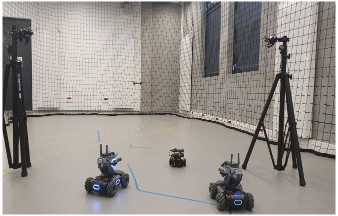

Fig. 2: Envisaged application

Moving Horizon Estimation

Moving Horizon Estimation (MHE):

- optimal state observer approach (duality with MPC)
- computing a state estimate by minimizing a cost function involving a plant model and a finite sequence of past measurements.

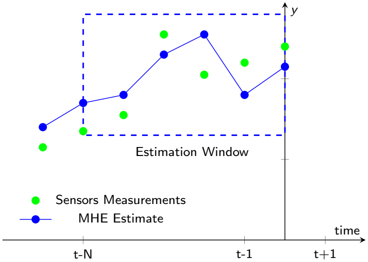

Fig. 3: Illustration of MHE

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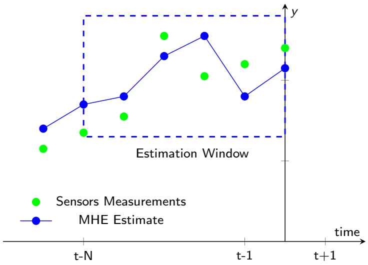

Fig. 3: Illustration of MHE

Compared to classical DSE (e.g., Distributed Kalman Filter DKF / Distributed Extended Kalman Filter DEKF)

Advantage:

- Account for constraints and non-linearities in its formulation.

Drawback:

- Large computation time due to an online optimization

Previous works on DMHE

- Distributed Moving Horizon Estimation approach:
  - Using neighborhood measurements, introducing different ways to fuse information, different consensus steps: consensus on the arrival cost [Farina et al. 2010], information-based consensus [Battistelli 2018].

    → Ensuring stability of the estimation error dynamics, under the assumptions of network connectivity and collective observability [Battistelli 2018].
- Reducing the computation burden:
  - Using a DMHE with a "pre-estimation" strategy based on a Luenberger observer [Venturino et al. 2020]

    → Empowering real-time implementation on low-cost processors.

    → Considering linear systems with **linear measurements**.

Motivations and objectives

- Motivations:
  - Increasingly recurrent use of low-cost embedded sensors (e.g., lidar, ultra-wideband mounted on mobile robots) providing nonlinear measurements (e.g., angle and/or distance)
- Objectives:

→ Developing DMHE algorithms with pre-estimation able to handle **nonlinear measurements**.

→ Ensuring the feasibility of a real-time application of this approach on low-cost processors.

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- Objectives:
  $\rightarrow$ Developing DMHE algorithms with pre-estimation able to handle **nonlinear measurements**.
  $\rightarrow$ Ensuring the feasibility of a real-time application of this approach on low-cost processors.
- First application:
  $\rightarrow$ Collaborative localization of a fleet of UAVs

MAS and communication modeling

- Considered Multi-Agent System (MAS): fleet of n_a UAVs, equipped with sensors, able to send and receive information through communication links with neighbors.
  $\mathcal{N}_a = \{1, 2, \ldots, n_a\}$: set of all the agents (nodes).
  $\mathcal{E} \subseteq \mathcal{N}_a \times \mathcal{N}_a$: set of all edges, communication links between agents.

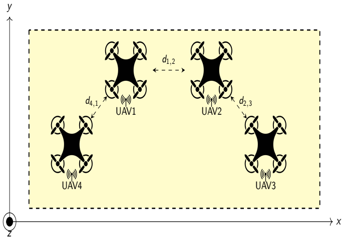

Fig. 4: Cooperative localization for a UAVs fleet

Remark 2.1

Information shared: either agent i measurements y^i provided by its embedded sensors, or its prior estimation of the entire MAS state $\hat{\mathbf{x}}^i$.

Assumption 1

Limited communication range $\rightarrow$ only one-step ("one-hop") neighbors communication considered.

- Communication network $\mathcal{G} = (\mathcal{N}_a, \mathcal{E})$: undirected connected graph.

Definition and notation

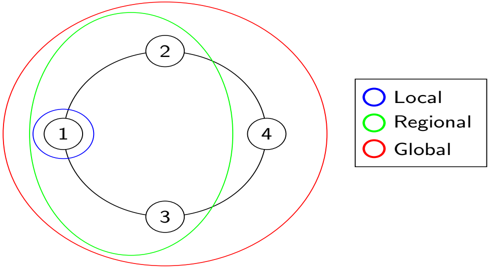

Fig. 5: Example of MAS communication network graph with 4 agents

For agent 1:

- *Local* information referring only to the local agent (e.g., x^1 its local state vector)
- *Regional* information referring to its one step neighborhood $\mathcal{N}_a^1$ (e.g., $\bar{y}^1$ its regional measurement)
- *Global* information considering the entire MAS (e.g., $\boldsymbol{x}$, the global MAS state vector)

- DSE for cooperative localization: each UAV uses regional information to estimate the state (position and speed) of each UAV of the entire MAS.

Dynamic modeling of the UAV and global MAS

Dynamics of each UAV $i \in \mathcal{N}_a$ of the MAS: discrete-time Linear Time-Invariant (LTI) model

$$x_{t+1}^i = A^i x_t^i + B^i u_t^i + B^i w_t^i \tag{1}$$

x^i: state vector u^i: input vector w^i: input noise vector, zero mean noise of covariance Q^i
A^i: evolution matrix B^i: input matrix
$u^i + w^i$: available input vector (noisy acceleration measurement from Inertial Measurement Unit).

Dynamics of the global MAS

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\boldsymbol{u}_t + \boldsymbol{B}\boldsymbol{w}_t \in \mathbb{R}^{n_x} \tag{2}$$

$\boldsymbol{x} = \text{col}(x^1, x^2, \dots, x^{n_a}) = [(x^1)^\top, (x^2)^\top, \dots, (x^{n_a})^\top]^\top$: collective (global) state
$\boldsymbol{u} = \text{col}(u^1, \dots, u^{n_a})$: collective input
$\boldsymbol{w} = \text{col}(w^1, \dots, w^{n_a})$: collective noise input
$\boldsymbol{A} = \text{diag}(A^1, \dots, A^{n_a})$
$\boldsymbol{B} = \text{diag}(B^1, \dots, B^{n_a})$

Assumption and measurement model

- Restrained communication:
  Each agent i does not communicate with the neighbors its available input vector $u^i + w^i$
  $\rightarrow$ Inputs of other agents are seen as unknown inputs by the agent i observer
  Global estimated input:

$$\hat{\boldsymbol{u}}^i = \mathrm{col}(0, \ldots, u^i + w^i, \ldots, 0) \in \mathbb{R}^{n_u} \tag{3}$$

Measurements locally performed by each agent i

$$y_t^i = h^i(\boldsymbol{x}_t) + \nu_t^i, \quad i \in \mathcal{N}_a \tag{4}$$

with the output vector y^i and the measurement noise $\nu^i \in \mathcal{V}^i$ of covariance R^i.

- Regional measurements:

$$\bar{y}_t^i = \mathrm{col}\left(y_t^i,\, y_t^{j_1}, \ldots, y_t^{j_{n_a^i}}\right), \; i \in \mathcal{N}_a \text{ and } [j_1, \ldots, j_{n_a^i}] \in \mathcal{N}_a^i \tag{5}$$

Remark 2.2

Nonlinear *dependence on the MAS state considered with the measurement function* h^i.

Local optimization problem of standard DMHE

At time t, each agent $i \in \mathcal{N}_a$ determines (based on regional information) its sequence of global MAS state estimate $[\hat{\boldsymbol{x}}^i_{t-N}, \dots, \hat{\boldsymbol{x}}^i_t]$ by solving:

Standard DMHE constrained optimization problem [Battistelli 2018; Farina et al. 2010]

$$\min_{[\hat{\boldsymbol{x}}^i_{t-N}, \dots, \hat{\boldsymbol{x}}^i_t]} J^i_N(\cdot) \tag{6}$$

$$\text{s.t.}\ \hat{\boldsymbol{x}}^i_k \in \mathcal{X}, \quad \bar{y}^i_{k+1} - \bar{h}^i(\hat{\boldsymbol{x}}^i_{k+1}) \in \bar{\mathcal{V}}^i, \tag{7}$$

$$\forall k = t-N, \dots, t-1.$$

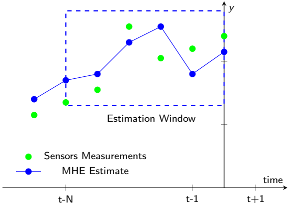

Fig. 6: Illustration of Moving Horizon Estimation (MHE)

N: estimation window length

Cost function

$$J^i_N(\cdot) = \sum_{k=t-N}^{t} \left\| \bar{y}^i_k - \bar{h}^i(\hat{\boldsymbol{x}}^i_k) \right\|^2_{(\bar{R}_i)^{-1}} + \sum_{k=t-N}^{t-1} \|\hat{\boldsymbol{x}}^i_{k+1} - A\hat{\boldsymbol{x}}^i_k - B\hat{\boldsymbol{u}}^i_k\|^2_{Q^{-1}} + \Gamma^i_t(\cdot) \tag{8}$$

Local optimization problem of DMHE with EKF pre-estimation

At time t, each agent $i \in \mathcal{N}_a$ determines (based on regional information) its global MAS state estimate $\hat{\boldsymbol{x}}^i_{t-N|t}$ of $\boldsymbol{x}_{t-N}$

DMHE with EKF pre-estimation constrained optimization problem

$$\min_{\hat{\boldsymbol{x}}^i_{t-N}} J^i_N(\cdot) \tag{9}$$

$$\text{s.t.}\, \hat{\boldsymbol{x}}^i_{k+1} = \hat{\boldsymbol{x}}^i_{k+1|k} + K^i_k \left(\bar{y}^i_{k+1} - \bar{h}^i(\hat{\boldsymbol{x}}^i_{k+1|k})\right), \tag{10}$$

$$\hat{\boldsymbol{x}}^i_{k+1|k} = \boldsymbol{A}\, \hat{\boldsymbol{x}}^i_k + \boldsymbol{B}\, \hat{\boldsymbol{u}}^i_k \tag{11}$$

$$\hat{\boldsymbol{x}}^i_k \in \mathcal{X}, \quad \bar{y}^i_{k+1} - \bar{h}^i(\hat{\boldsymbol{x}}^i_{k+1}) \in \bar{\mathcal{V}}^i, \tag{12}$$

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Pre-estimation EKF (with classical gain update)

- Using the pre-estimation observer → reconstruct $[\hat{\boldsymbol{x}}^i_{t-N}, \ldots, \hat{\boldsymbol{x}}^i_t]$ and keep $\hat{\boldsymbol{x}}^i_t$.
- Compared to standard DMHE, less optimization variables → reduced computation time [Venturino et al. 2021]

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L-step information consensus

Penalty function / arrival cost

$$\Gamma_t^i(.) = \|\hat{\mathbf{x}}_{t-N}^i - \bar{\mathbf{x}}_{t-N}^i\|^2_{(\tilde{\Pi}_{t-N}^i)^{-1}} \quad (14)$$

A priori state estimate and weight matrix obtained with a L-step consensus on information [Battistelli 2018]

1. Initialization:

$$P_{t-N,0}^i = (\Pi_{t-N}^i)^{-1} \quad \text{(information matrix)}$$

$$\xi_{t-N,0}^i = P_{t-N,0}^i \hat{\mathbf{x}}_{t-N|t-1}^i \quad \text{(information vector)}$$

2. Consensus step: exchanging $P_{t-N,0}^i$ and $\xi_{t-N,0}^i$ with neighbors then compute the weighted average.

$$P_{t-N,l+1}^i = k_{i,i} P_{t-N,l}^i + \sum_{j \in \mathcal{N}_a^i} k_{i,j} P_{t-N,l}^j \quad (15)$$

$$\xi_{t-N,l+1}^i = k_{i,i} \xi_{t-N,l}^i + \sum_{j \in \mathcal{N}_a^i} k_{i,j} \xi_{t-N,l}^j \quad (16)$$

with $l \in \{0, \ldots, L-1\}$

3. After L-steps of consensus, the arrival cost (14) uses

$$\tilde{\Pi}_{t-N}^i = (P_{t-N,L}^i)^{-1} \quad (17)$$

$$\bar{\mathbf{x}}_{t-N}^i = (P_{t-N,L}^i)^{-1} \xi_{t-N,L}^i \quad (18)$$

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- Advantages: allow to broadcast information deeper and take into account the confidence of each agent on each component of the state.

Observability rank-based weights technique

- Observability rank-based weights $k_{i,j}$ in consensus steps → extension of [Venturino et al. 2022] to the nonlinear case

Regional observability matrix of agent i over the estimation window $[t-N,t]$

$$\bar{\mathcal{O}}^i_{N,t} = \left[(\bar{C}^i_{t-N})^\top \quad (\bar{C}^i_{t-N+1}\boldsymbol{A})^\top \quad \cdots \quad (\bar{C}^i_t \boldsymbol{A}^N)^\top \right]^\top, \qquad \text{with} \;\; \bar{C}^i_{t-N} = \left. \frac{\partial \, \bar{h}^i}{\partial \boldsymbol{x}} \right|_{\hat{\boldsymbol{x}}^i_{t-N}} \tag{19}$$

- $\rho^i_{\mathcal{O}}$: reliability of agent i, exchanged with neighbors

$$\rho^i_{\mathcal{O}} = \text{rank}(\bar{\mathcal{O}}^i_{N,t}) \tag{20}$$

- $k_{i,j}$: ratio of $\rho^i_{\mathcal{O}}$ averaged among the neighbors $j \in \mathcal{N}^i_a$

$$k_{i,j} = \frac{\rho^j_{\mathcal{O}}}{\sum_{l \in \mathcal{N}^i_a} \rho^l_{\mathcal{O}}}. \tag{21}$$

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- Method suitable for distributed schemes → mitigate unobservability issues

Simulation cooperative localization of an UAVs fleet

Assumptions:

- Communication network: undirected time-invariant connected graph
- No communication failure

Considered system:

- Fleet of $n_a = 3$ UAVs
- Linear dynamics
- Non linear measurements (distance between neighbors UAVs and speed norm)

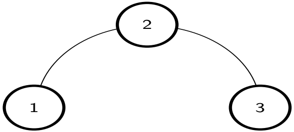

Fig. 7: Communication graph and distance measurement capabilities between the three UAVs

Results on the constrained cooperative localization of an UAVs fleet

Comparison

- **DMHE-pre-EKF**: our proposed DMHE approach with EKF pre-estimation
- **DMHE-1**: standard DMHE algorithm [Battistelli 2018] (without pre-estimation) extended to nonlinear measurements
- **DEKF-CI**: consensus-based on information distributed EKF of [Battistelli et al. 2015]
- **DMHE-pre-EKF-2-step**: our DMHE-pre-EKF observer with 2-step consensus (i.e., $L = 2$)

	RMSE	RMSE final values	τ (s)
DEKF-CI	2.7651	0.7059	0.0004
DMHE-1	1.6371	0.6906	0.4946
DMHE-pre-EKF	1.7009	0.6104	0.0413
DMHE-pre-EKF-2-step	1.6709	0.5980	0.0433

Tab. 1: Comparative results of the different estimation techniques

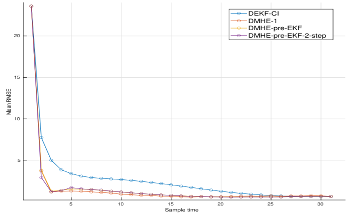

Fig. 8: Averaged RMSE among all the agents and all the trials

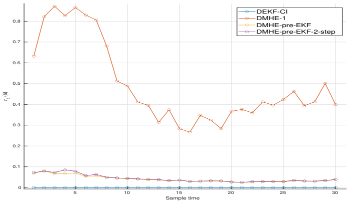

Fig. 9: Averaged computation time τ (s) among all the agents and all the trials

Conclusion

Concluding remarks

- Contribution: DMHE algorithm with EKF-based pre-estimation for constrained cooperative localization of a Multi-Agent System with nonlinear measurements.
- Simulation results confirm the interest of the proposed method: constraints handling, reducing computation load, preserving estimation accuracy (compared to standard DMHE).

Associated paper

- **M. Borelle**, S. Bertrand, C. Stoica, T. Alamo, E.F. Camacho, "Cooperative localization of an UAV fleet using distributed MHE with EKF pre-Estimation and nonlinear measurements", 27th International Conference on System Theory, Control and Computing, Timisoara, Romania, 2023.

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Current work

- Practical implementation of the proposed approach on a real MAS composed of UAVs, UGVs (flight arena of CentraleSupélec).

Thanks for your attention !

Questions ?

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Prior Π^i weight update

- The arrival cost is designed to approximate the untractable full information problem (that processes all the information from initial time).
- When the system is non-linear and constrained, an algebraic update expression for the arrival cost rarely exists. $\rightarrow$ approximate the arrival cost for the constrained problem with the one of the unconstrained problem [Rao et al. 2003].

Thus, the positive definite matrix Π^i_{t-N+1} is obtained from the matrix $\tilde{\Pi}^i_{t-N}$ using the discrete-time Riccati equation associated to an Extended Kalman filter (as in [Rao et al. 2003] for the centralized case):

$$\Pi^i_{t-N+1|t-N} = \boldsymbol{A}\tilde{\Pi}^i_{t-N}\boldsymbol{A}^\top + \boldsymbol{B}\boldsymbol{Q}^i\boldsymbol{B}^\top \tag{22}$$

$$\Pi^i_{t-N+1} = \Pi^i_{t-N+1|t-N} - \Pi^i_{t-N+1|t-N}(\bar{C}^i_{t-N+1})^\top \left(\bar{C}^i_{t-N+1}\Pi^i_{t-N+1|t-N}(\bar{C}^i_{t-N+1})^\top + \bar{R}^i\right)^{-1} \bar{C}^i_{t-N+1}\Pi^i_{t-N+1|t-N} \tag{23}$$

with $\boldsymbol{Q}^i = \mathrm{diag}(Q^1, \ldots, Q^{n_a})$

EKF pre-estimation observer

For $k \in \{t-N,\dots,t\}$, the matrix gain K_k^i is computed using the classical EKF observer update. The step $t-N$ consists of initializing the pre-estimation error covariance matrix $\Pi^i_{pre,t-N|t-N} = \tilde{\Pi}^i_{t-N}$. Then, for $k \in \{t-N+1,\dots,t\}$, the prediction of the covariance evolution is performed as follows:

$$\Pi^i_{pre,k|k-1} = \boldsymbol{A}\Pi^i_{pre,k-1|k-1}\boldsymbol{A}^\top + \boldsymbol{B}\boldsymbol{Q}^i\boldsymbol{B}^\top \tag{24}$$

avec $\Pi^i_{pre,k|k-1}$ the a priori estimation matrix of covariance of the estimation error at time k.
The optimal Kalman gain (10) is computed as follows:

$$K_k^i = \Pi^i_{pre,k|k-1}(\bar{C}^i_k)^\top (S_k^i)^{-1} \tag{25}$$

with the pre-estimation error covariance matrix:

$$\Pi^i_{pre,k|k} = (I_{n_x} - K_k^i\,\bar{C}^i_k)\Pi^i_{pre,k|k-1} \tag{26}$$

and S_k^i the innovation covariance:

$$S_k^i = \bar{C}^i_k\Pi^i_{pre,k|k-1}(\bar{C}^i_k)^\top + \bar{R}^i \tag{27}$$

Remark: to ensure robustness of the estimation, taking into account the uncertainties on the input measurement of the other agents, we artificially increase the corresponding diagonal element of $\boldsymbol{Q}^i$.

Proposed DMHE with EKF pre-estimation algorithm

Algorithm 1 DMHE with pre-estimation procedure Part 1

1: **Initialization:** $\forall i \in \mathcal{N}_a$, at the first time step $t = 0$
2: **initialize** Π_0^i, $\hat{\boldsymbol{x}}_0^i$
3: **collect** a first local measurement y_0^i and the knowledge on the initial collective input $\hat{\boldsymbol{u}}_0^i$
4: **receive** from the neighborhood $j \in \mathcal{N}_a^i$ their measurements y_0^j
5: **Online:** $\forall i \in \mathcal{N}_a$, $\forall t > 0$
6: **collect** the local measurement y_t^i and the knowledge on the collective input $\hat{\boldsymbol{u}}_t^i$ using (4) and (3)
7: **receive** from the neighbors $j \in \mathcal{N}_a^i$ the collected measurements in the step 6, form and store $\bar{y}_t^i$
8: **if** $1 \leqslant t \leqslant N$ **then**
9: **set** the horizon length $N_w = t$
10: **else**
11: **set** the horizon length $N_w = N$
12: **end if**
13: **compute** $\mathcal{O}_{N_w,t}^i$ and $\rho_{\mathcal{O}}^i$ according to (19) and (20)
14: **exchange** $\rho_{\mathcal{O}}^j$ with $j \in \mathcal{N}_a^i$
15: **compute** the k_{ij} components according to (21)
16: **perform** L steps of the consensus algorithms (16) with the initialization to get $\tilde{\Pi}_{t-N_w}^i$ and $\bar{\boldsymbol{x}}_{t-N_w}^i$ (17)-(18)

Proposed DMHE with EKF pre-estimation algorithm

Algorithm 2 DMHE with pre-estimation procedure Part 2

```
solve the local optimization problem of DMHE with EKF pre-estimation, minimizing $J^i_{N_w}$ as in (13) and (14) subject to the constraints (10)-(12)
    store the solution $\hat{x}^i_{t-N_w|t}$, $\hat{x}^i_{t-N_w+1|t}$ and the corresponding estimate $\hat{x}^i_{t|t}$
    if $1 \leqslant t \leqslant N$ then
        set $\bar{x}^i_{0|t+1} = \hat{x}^i_{0|t}$
        set $\Pi^i_{0|t+1} = \tilde{\Pi}^i_{0|t}$
    else
        compute $\Pi^i_{t+1-N}$ according to (22)-(23)
        compute prediction $\bar{x}^i_{t+1-N} = \hat{x}^i_{t+1-N|t}$
    end if
```