Nonlinear MPC for collision-avoidance trajectory tracking of the multi-UAV system in a mapping mission

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SUMMARY

1. Context
2. Mission planning for mapping
3. Prioritized trajectory tracking
4. Distributed NMPC strategies for trajectory tracking with collision avoidance
5. Robustness assessment
6. Conclusion and perspectives

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1. Context

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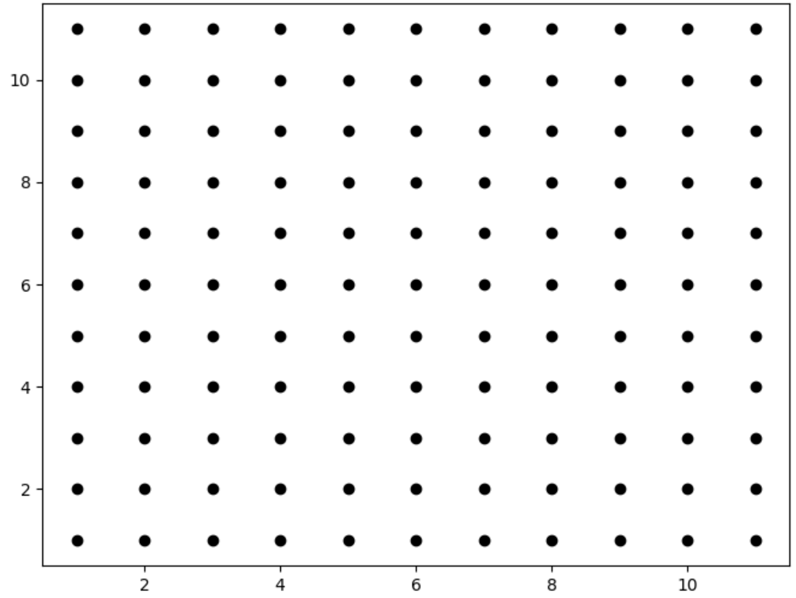
UAVs for mapping tasks in agriculture

- Growing potential of **unmanned aerial vehicles (UAVs)** in smart agriculture
  - Farming management optimization
  - Increased agricultural productivity
- Multi-UAV system for **remote sensing of the crops – mapping**
  - Increased mission efficiency
  - Reduced mapping duration
- Mapping mission consists of:
  1. **Mission planning**
  2. **Trajectory tracking**

2. Mission planning for mapping

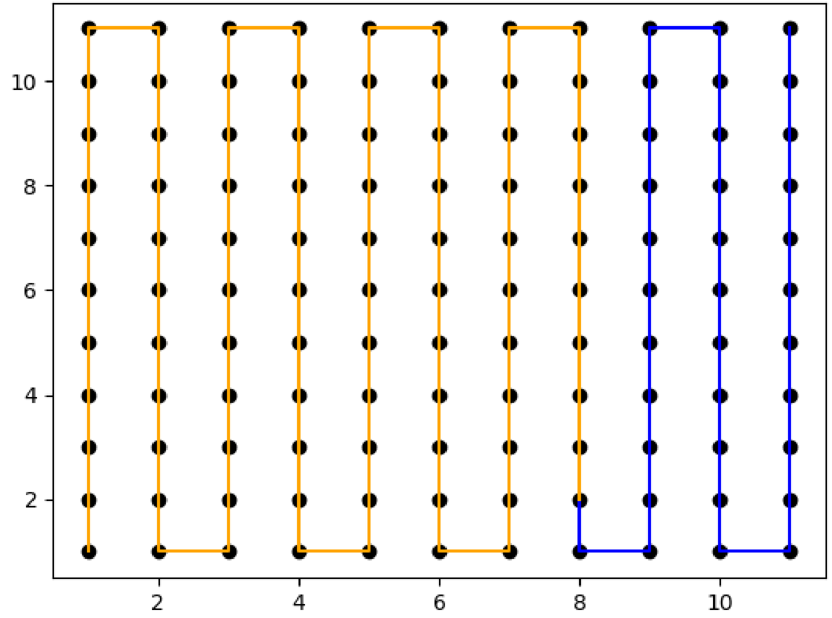
Mission planning for a multi-UAV system

- Mission planning implies:
  - **Area decomposition** – considering field shape, UAV and camera characteristics



Mission planning for a multi-UAV system

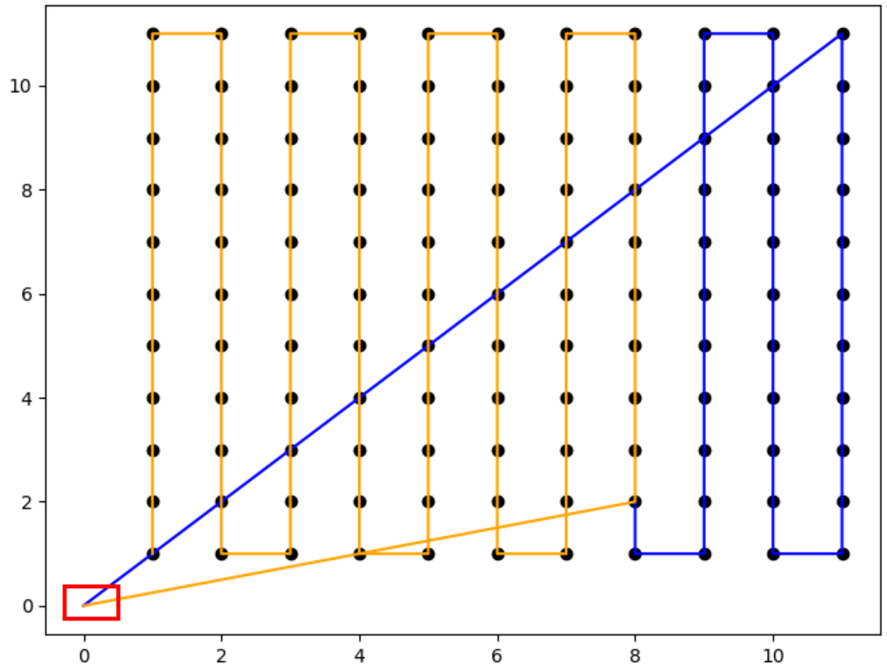
- Mission planning implies:
  - **Area decomposition** – considering field shape, UAV and camera characteristics
  - **Task allocation** – distribution of the waypoints



Mission planning for a multi-UAV system

- Mission planning implies:
  - **Area decomposition** – considering field shape, UAV and camera characteristics
  - **Task allocation** – distribution of the waypoints
  - **Energy management** – battery replacement strategy

Example of path planning optimization for battery management



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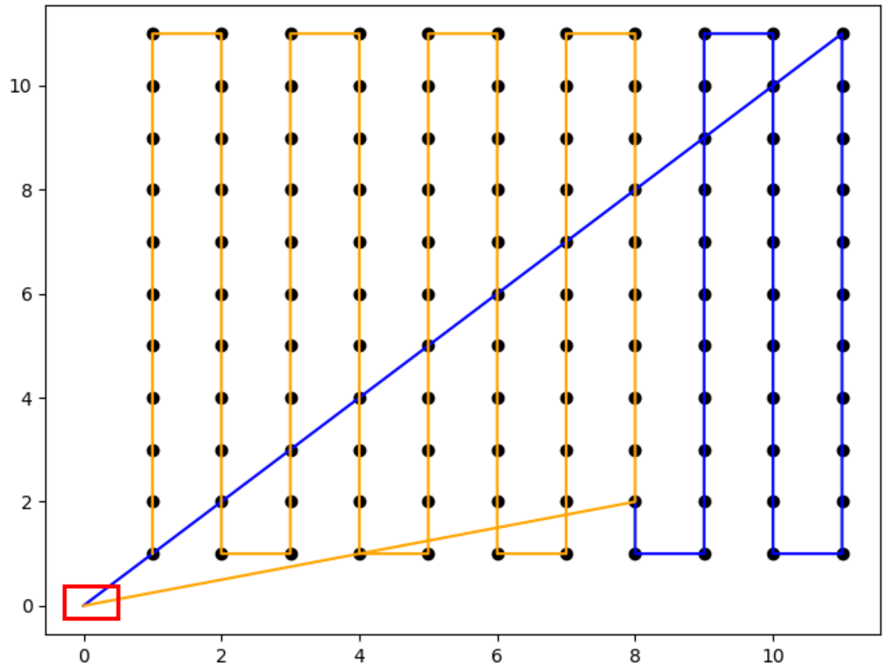
Multi-UAV mapping mission

- Challenges:
  - **Energy-aware mission**
    - battery usage
    - safe return-to-base in case of insufficient energy
  - **Cooperative multi-UAV system**
    - Task distribution and allocation
    - Collision avoidance

Example of path planning optimization for battery management



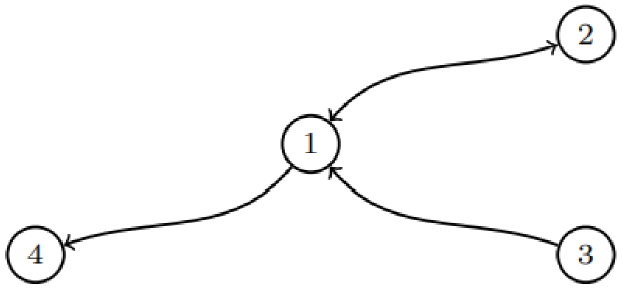
3. Prioritized trajectory tracking

Trajectory tracking

- Suitable control strategy:
  - **Minimizes tracking error**
  - **Robust against disturbances**
- Challenges:
  - Often irregular shape of the field → **nonlinear optimal trajectory**
  - Multi-UAV system → **coordination, collision avoidance**
- Promising results: Model predictive control (MPC)
  - Ability to handle constraints
  - Multi-UAV system → **distributed** approach

Trajectory tracking with collision avoidance

- Multi-UAV mission
- Defined 3D reference trajectory for each UAV
- Objective:
  - Track the reference trajectory for each UAV while avoiding collision
- **Approach:**
  - **Distributed nonlinear MPC** with full information exchange between UAVs



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Prioritized trajectory tracking

- **Passing priority allocation** - Predetermined hierarchy depending on defined criteria → flight duration, battery level, etc.
  - UAV with the **higher-passing priority**
    - Classical NMPC – reference trajectory tracking
  - UAV with **lower-passing priority**
    - NMPC with collision avoidance

  → Redundant maneuvers elimination

  → Minimize the path alterations for the leading UAV

  - **Optimal control problem with reduced computational complexity**

4. Distributed NMPC strategies for trajectory tracking with collision avoidance

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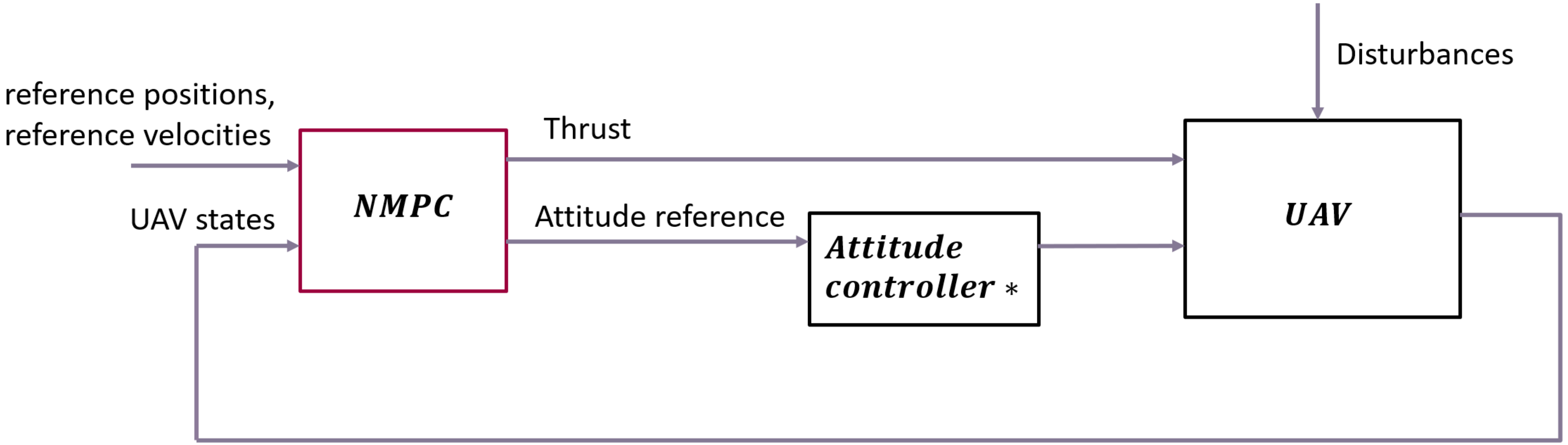
Distributed NMPC for collision avoidance

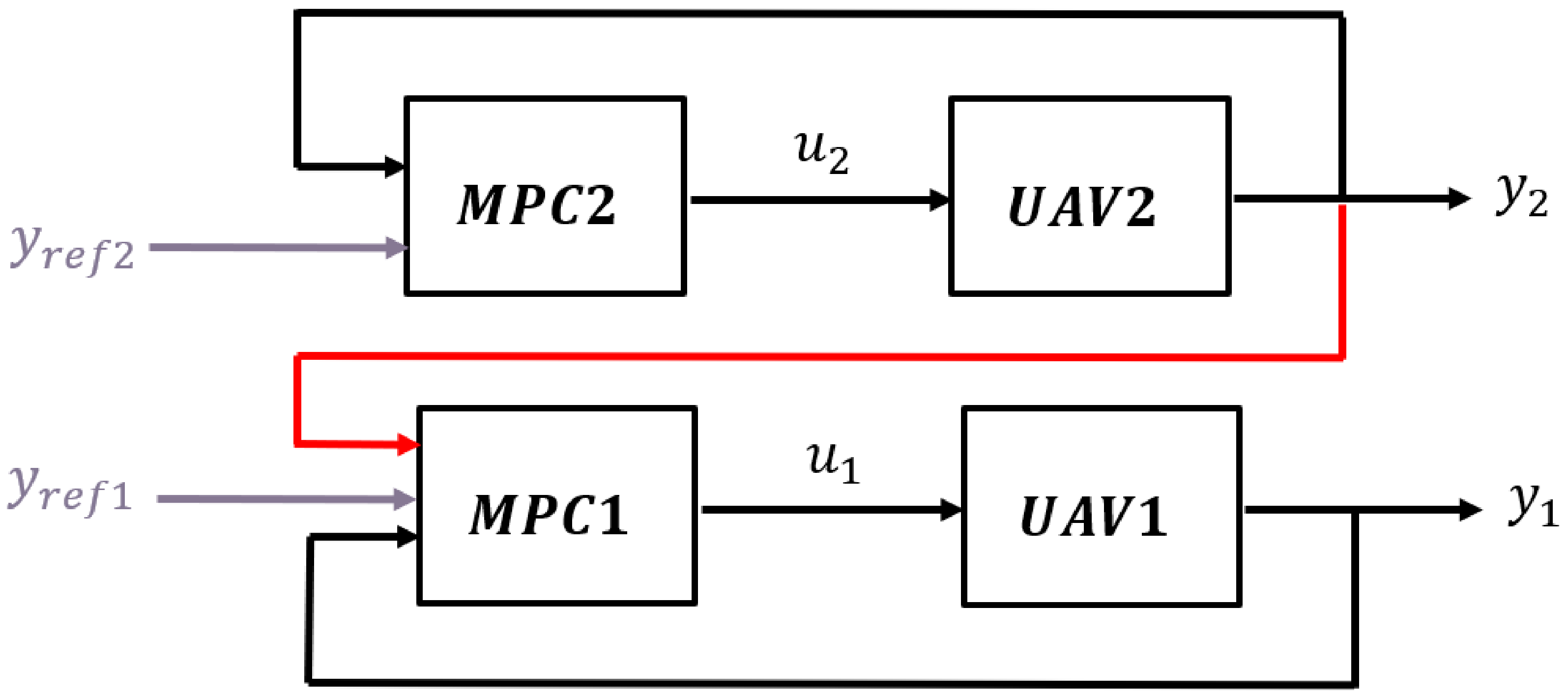
- Control architecture for each UAV:



* Lindqvist, B., Mansouri, S., Agha-mohammadi, A. A., Nikolakopoulos, G., Nonlinear MPC for collision avoidance and control of UAVs with dynamic obstacles. IEEE robot. Autom. Lett., 5(4), 2020.

Distributed NMPC for collision avoidance

- Prioritized tracking with collision avoidance for a 2-UAV system

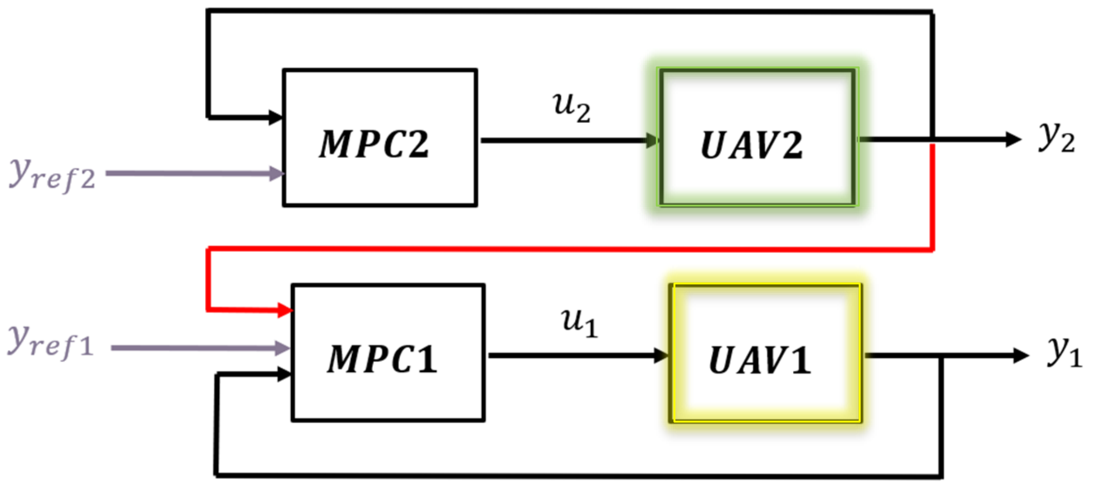


Higher-passing priority UAV

Lower-passing priority UAV

Distributed NMPC for collision avoidance

- **Prioritized tracking with collision avoidance for a 2-UAV system**
  - UAV 2 – higher passing priority
    - Classical NMPC
  - UAV 1 – lower passing priority
    - NMPC with **collision avoidance**:
      1. As a **nonlinear constraint**
      2. In the **cost function**
      3. Through a **flight corridor**



0. Classical NMPC

- Cost function: $$\min_{u^i_{k,\dots,k+N_p-1}} J_i\,, \qquad i \in N_i$$

$u^i_{k,\dots,k+N_p-1}$ → *Future control inputs*

$$J_i\left(u^i_{k,\dots,k+N_p-1}\right) = \underbrace{\sum_{n=1}^{N_p} \left\| \hat{y}^i_{k+n} - y^{i,ref}_{k+n} \right\|^2_{Q_i}}_{\text{output cost}} + \underbrace{\left\| \Delta u^i_{k+n-1} \right\|^2_{R_i}}_{\text{input cost}}$$

output cost → ***Tracking error***

input cost → ***Control smoothness***

Subject to: $u \in \mathcal{U}$

N_i	:	set of UAVs in the system
Q_i, R_i	:	weighting matrices for the UAV i
$\hat{y}^i_{k+n}$	:	predicted output of the UAV i
$y^{i,ref}_{k+n}$	:	reference output of the UAV i
Δu^i_{k+n-1}	:	change in successive control inputs of the UAV i

1. Collision avoidance as a nonlinear constraint

- Cost function: $\min\limits_{u^i_{k,\dots,k+N_p-1}} J_i\,, \quad i \in N_i$ → *Future control inputs*

$$J_i\left(u^i_{k,\dots,k+N_p-1}\right) = \underbrace{\sum_{n=1}^{N_p} \left\| \hat{y}^i_{k+n} - y^{i,ref}_{k+n} \right\|^2_{Q_i}}_{\text{output cost}} + \underbrace{\left\| \Delta u^i_{k+n-1} \right\|^2_{R_i}}_{\text{input cost}}$$

output cost → ***Tracking error***

input cost → ***Control smoothness***

Subject to: $u \in \mathcal{U}$

$$\left\| d_{ij} \right\|_2 \geq d_s$$

→ ***Collision avoidance***

N_i	:	set of UAVs in the system
Q_i, R_i	:	weighting matrices for the UAV i
$\hat{y}^i_{k+n}$	:	predicted output of the UAV i
$y^{i,ref}_{k+n}$	:	reference output of the UAV i
Δu^i_{k+n-1}	:	change in successive control inputs of the UAV i
d_{ij}	:	distance between the position of the UAV i and UAV j
d_s	:	safety distance

2. Collision avoidance in the cost function

- Cost function: $\min_{u^i_{k,\dots,k+N_p-1}} J_i, \quad i \in N_i$ → *Future control inputs*

$$J_i\left(u^i_{k,\dots,k+N_p-1}\right) = \underbrace{\sum_{n=1}^{N_p} \left\| \hat{y}^i_{k+n} - y^{i,ref}_{k+n} \right\|^2_{Q_i}}_{\text{output cost}} + \underbrace{\left\| \Delta u^i_{k+n-1} \right\|^2_{R_i}}_{\text{input cost}} - \underbrace{\sum_{j \in N_i, j \neq i} \boldsymbol{A_{ij}} \left\| d_{ij,k+n} \right\|^2_{G_{ij}}}_{\text{(state-dependent) collision cost}}$$

Tracking error (output cost) — ***Control smoothness*** (input cost) — ***Collision avoidance*** (collision cost)

Subject to: $u \in \mathcal{U}$

N_i	:	set of UAVs in the system
Q_i, R_i, G_{ij}	:	weighting matrices for the UAV i
$\hat{y}^i_{k+n}$	:	predicted output of the UAV i
$y^{i,ref}_{k+n}$	:	reference output of the UAV i
Δu^i_{k+n-1}	:	change in successive control inputs of the UAV i
A_{ij}	:	2^{nd} criterion weight, can take the value in the interval $[0,1]$
$d_{ij,k+n}$	:	distance between the position of the UAV i and UAV j
d_s	:	safety distance

2. Collision avoidance in the cost function

- $\boldsymbol{A_{ij}} \in [\mathbf{0}, \mathbf{1}]$
  - Determines how strongly must the UAV avoid its neighbour
    - Depends on the distance between the two UAVs:

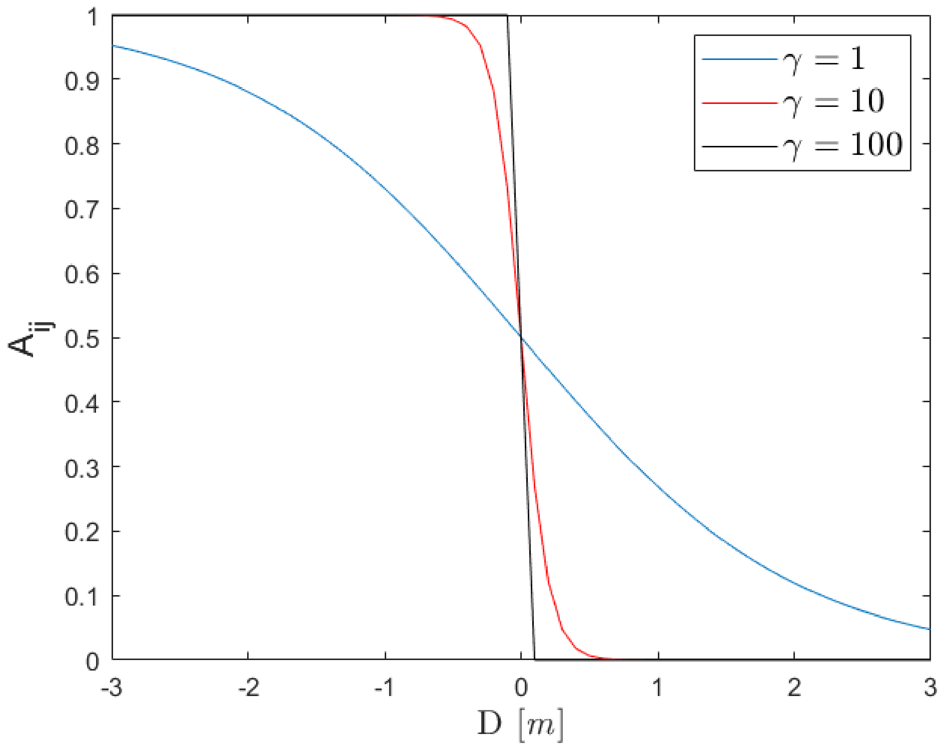
$$A_{ij} = \frac{1}{1 + e^{\gamma \cdot D}},$$

Where

$$D = d_{ij} - d_{min},$$

$$d_{min} = d_s \cdot (1 + S)$$

d_s : safety distance
d_{min} : minimum safety distance



3. Collision avoidance through a flight corridor

- Cost function: $\min_{u^i_{k,\dots,k+N_p-1}} J_i \,, \quad i \in N_i$ — Future control inputs

$$J_i\left(u^i_{k,\dots,k+N_p-1}\right) = \underbrace{\sum_{n=1}^{N_p} \left\| \hat{y}^i_{k+n} - y^{i,ref}_{k+n} \right\|^2_{Q_i}}_{\text{output cost}} + \underbrace{\left\| \Delta u^i_{k+n-1} \right\|^2_{R_i}}_{\text{input cost}}$$

Tracking error (output cost) — **Control smoothness** (input cost)

Subject to: $u \in \mathcal{U}$

$$\left\| p_x - p_x^{ref} \right\|_2 \le \beta,$$

$$\left\| p_y - p_y^{ref} \right\|_2 \le \beta,$$

$$\left\| p_z - p_z^{ref} \right\|_2 \le \beta,$$

→ **Flight corridor**

p_x	:	position of the UAV in x-axis
p_y	:	position of the UAV in x-axis
p_z	:	position of the UAV in x-axis
β	:	corridor width in 3 axes

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5. Robustness assessment

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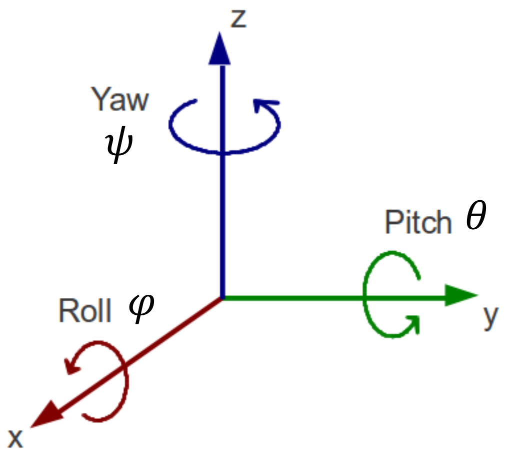
System dynamics – UAV model [1]

- States, controls and outputs: $\mathbf{x} = [p, v, \varphi, \theta]^T$, $\mathrm{u} = \left[T, \varphi_{ref}, \theta_{ref}\right]^T$, $\boldsymbol{y} = [p, v]^T$
  - *8 state variables:*
    - *Position*: $p = [x_c, y_c, z_c]^T$
    - *Velocities*: $v = [\dot{x}_c, \dot{y}_c, \dot{z}_c]^T$
    - *Roll and pitch*: φ, θ
    - *Yaw angle is set to zero, ψ=0 [1]*
  - *3 control inputs:*
    - *Thrust*: T
    - *Reference roll and pitch*: $\varphi_{ref}, \theta_{ref}$
  - *6 output variables:*
    - *Position*: $p = [x_c, y_c, z_c]^T$
    - *Velocities*: $v = [\dot{x}_c, \dot{y}_c, \dot{z}_c]^T$



[1] Lindqvist, B., Mansouri, S., Agha-mohammadi, A. A., Nikolakopoulos, G., Nonlinear MPC for collision avoidance and control of UAVs with dynamic obstacles. IEEE robot. Autom. Lett., 5(4), 2020.

System dynamics – UAV model [1]

- Dynamical nonlinear model of a quadrotor:

  - $\dot{p}(t) = v(t)$
  - $\dot{v}(t) = R\begin{bmatrix}0\\0\\\alpha T\end{bmatrix} + \begin{bmatrix}0\\0\\-g\end{bmatrix} - \begin{bmatrix}A_x & 0 & 0\\0 & A_y & 0\\0 & 0 & A_z\end{bmatrix}v(t) + \begin{bmatrix}w_x\\w_y\\w_z\end{bmatrix}$
  - $\dot{\varphi}(t) = (K_\varphi \varphi_{ref}(t) - \varphi(t))/\tau_\varphi$
  - $\dot{\theta}(t) = (K_\theta \theta_{ref}(t) - \theta(t))/\tau_\theta$

A_x, A_y, A_z : linear damping terms

K_φ, K_θ : gains (inner-loop control)

$\tau_\varphi, \tau_\theta$: time constants

Rotational matrix: $(\psi = 0)$

$$R = \begin{bmatrix} c\theta c\psi & s\varphi s\theta c\psi - c\varphi s\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ c\theta s\psi & s\varphi s\theta s\psi + c\varphi c\psi & c\varphi s\theta s\psi - s\varphi c\psi \\ -s\theta & s\varphi c\theta & c\varphi c\theta \end{bmatrix}$$

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System dynamics – UAV model [1]

- Dynamical nonlinear model of a quadrotor:

external disturbances

- $\dot{p}(t) = v(t)$
- $\dot{v}(t) = R\begin{bmatrix} 0 \\ 0 \\ \alpha T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} - \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix} v(t) + \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$

thruster efficiency

- $\dot{\varphi}(t) = (K_\varphi \varphi_{ref}(t) - \varphi(t))/\tau_\varphi$
- $\dot{\theta}(t) = (K_\theta \theta_{ref}(t) - \theta(t))/\tau_\theta$

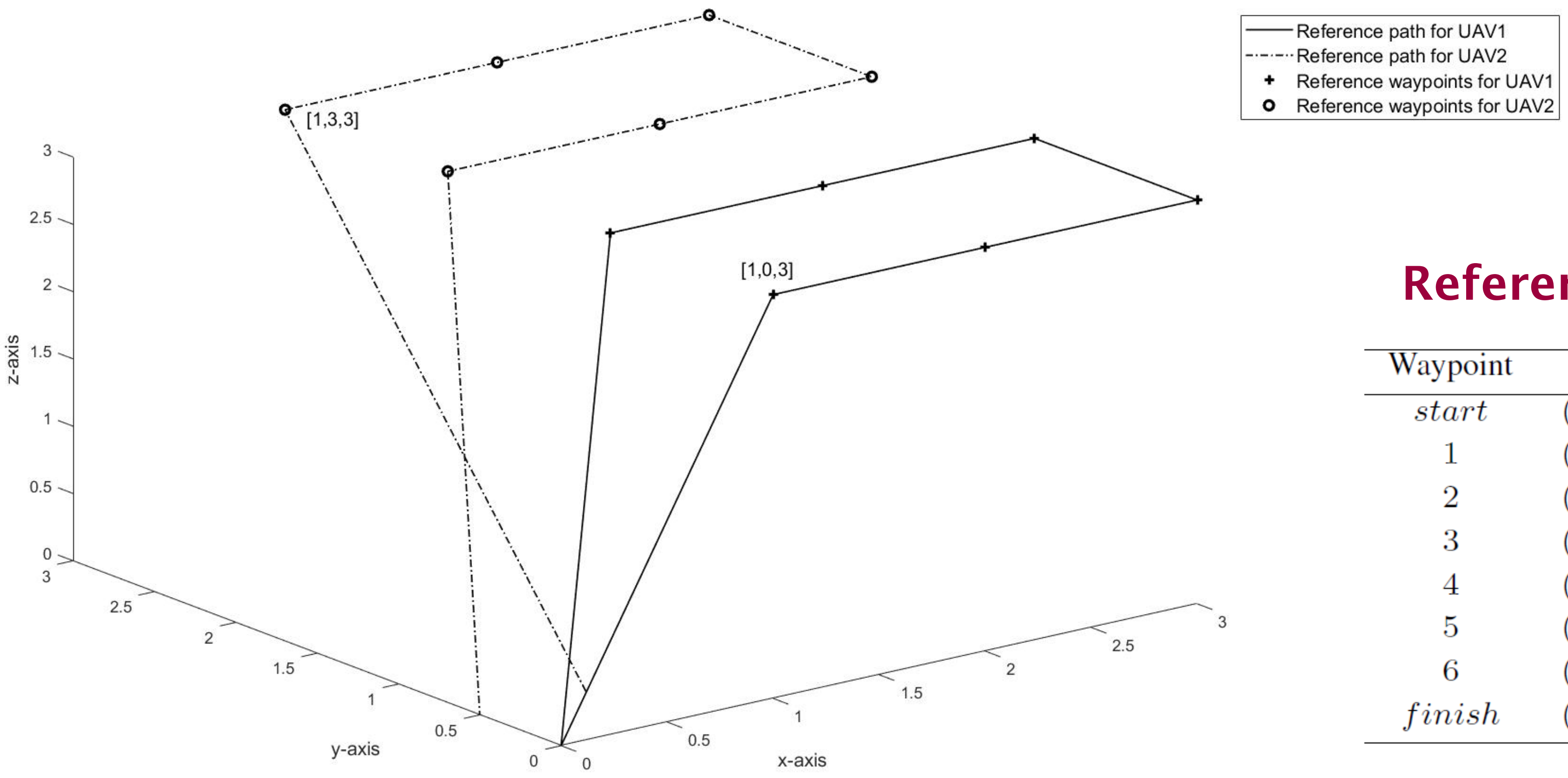
A_x, A_y, A_z	:	linear damping terms
K_φ, K_θ	:	gains (inner-loop control)
$\tau_\varphi, \tau_\theta$	:	time constants

Rotational matrix: $(\psi = 0)$

$$R = \begin{bmatrix} c\theta c\psi & s\varphi s\theta c\psi - c\varphi s\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ c\theta s\psi & s\varphi s\theta s\psi + c\varphi c\psi & c\varphi s\theta s\psi - s\varphi c\psi \\ -s\theta & s\varphi c\theta & c\varphi c\theta \end{bmatrix}$$

[1] Lindqvist, B., Mansouri, S., Agha-mohammadi, A. A., Nikolakopoulos, G., Nonlinear MPC for collision avoidance and control of UAVs with dynamic obstacles. IEEE robot. Autom. Lett., 5(4), 2020.

Mapping mission simulations – reference paths



Reference waypoints

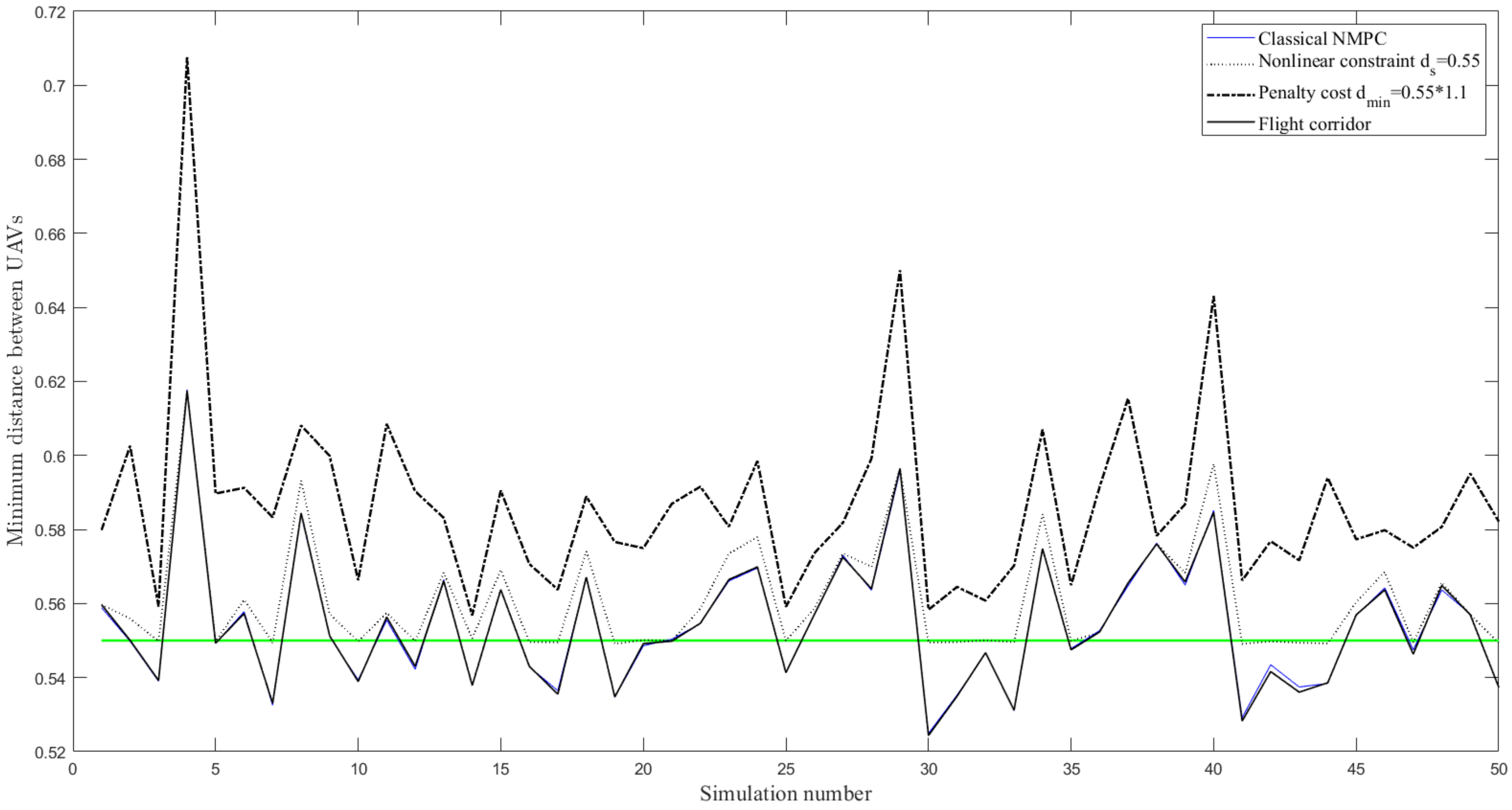
Waypoint	UAV1	UAV2
$start$	$(0,0,0)m$	$(0.5,0.5,0)m$
1	$(1,0,3)m$	$(1,3,3)m$
2	$(2,0,3)m$	$(2,3,3)m$
3	$(3,0,3)m$	$(3,3,3)m$
4	$(3,1,3)m$	$(3,2,3)m$
5	$(2,1,3)m$	$(2,2,3)m$
6	$(1,1,3)m$	$(1,2,3)m$
$finish$	$(0,0,0)m$	$(0,0.5,0)m$

Monte Carlo simulations – 50 test cases

- **Robustness assessment**
  - Random constant external disturbances $[w_x, w_y, w_z]$
  - Random uncertainty of the thruster efficiency parameter α
  - Safety distance $d_s = 0.55\,m$; Security factor $S = 10\%$
  - Corridor width $\beta = 0.3\,m$
- Solving the optimization problem: fmincon (Matlab)

Monte Carlo simulations – 50 test cases

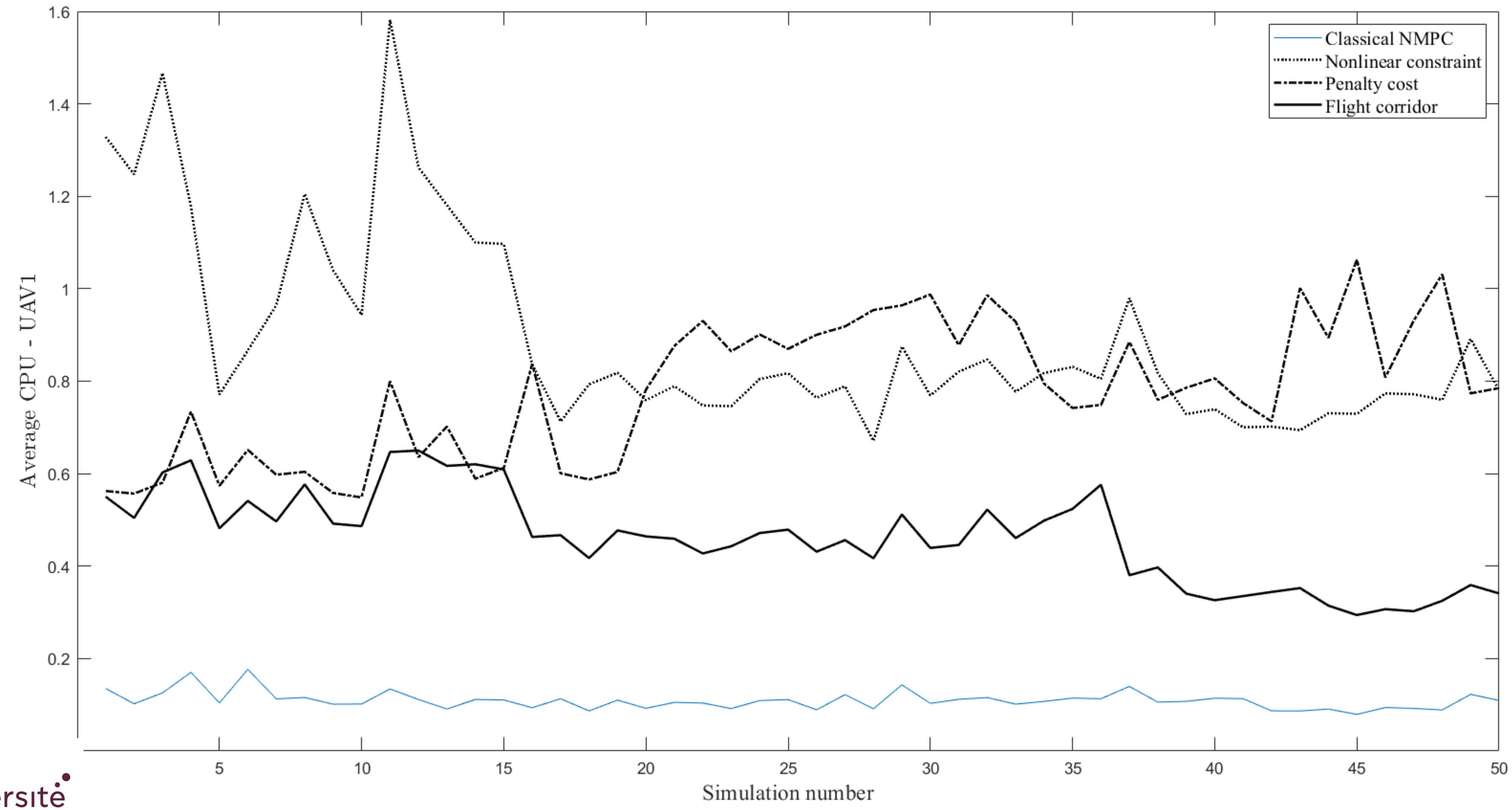
Minimum distance between UAVs



$d_s = 0.55\ m$

Monte Carlo simulations – 50 test cases

Average CPU



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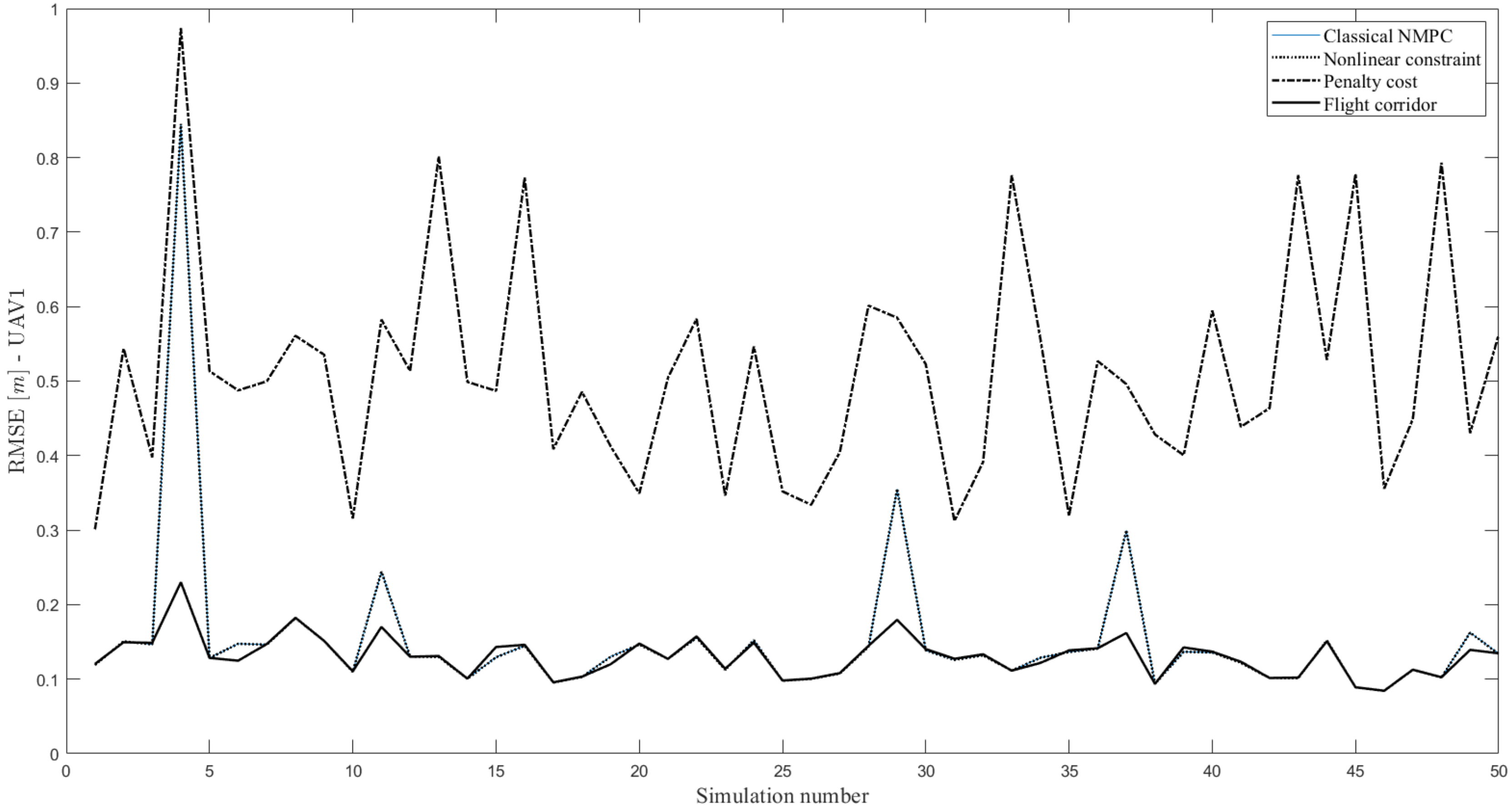
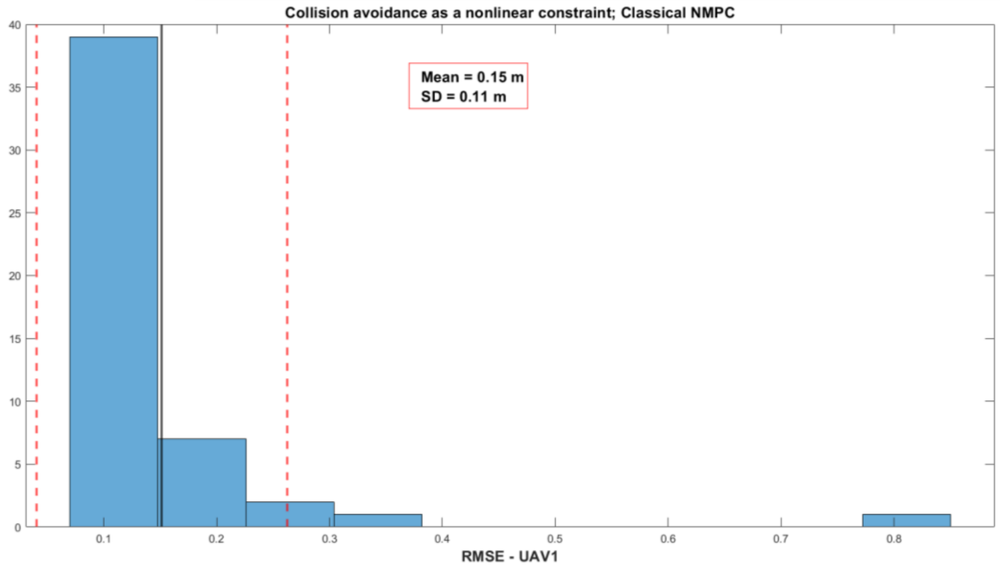
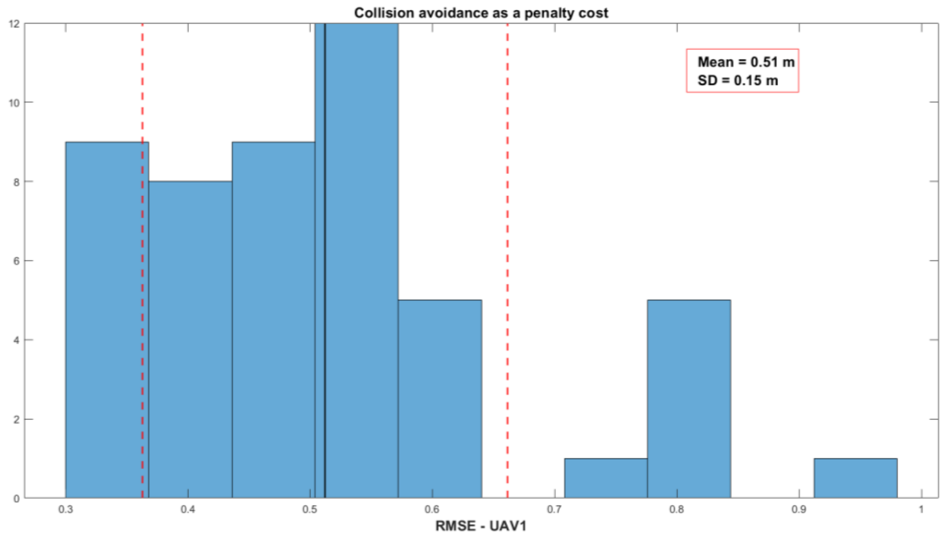
Monte Carlo simulations – 50 test cases

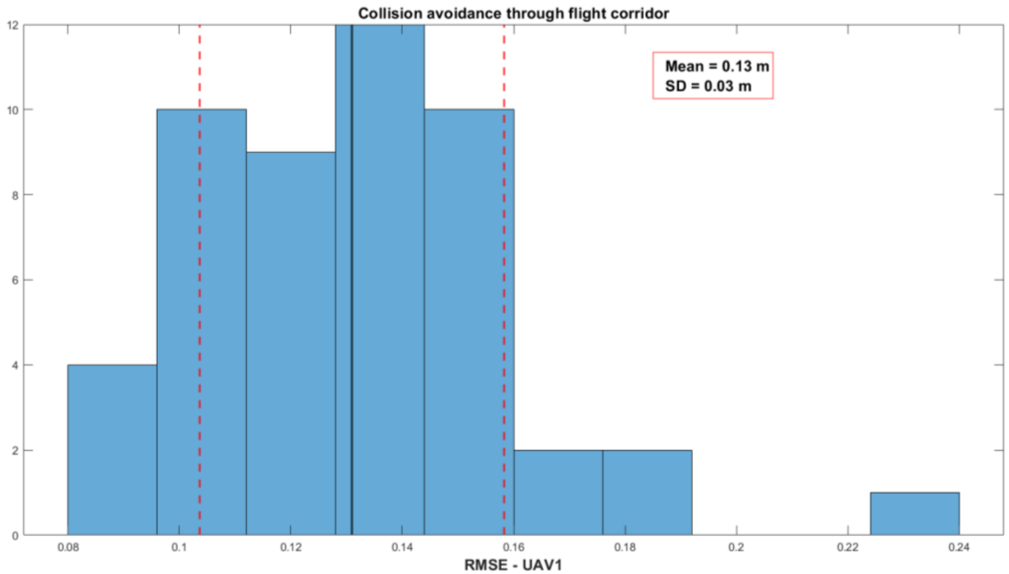
Root Mean Square Error



Monte Carlo simulations – 50 test cases

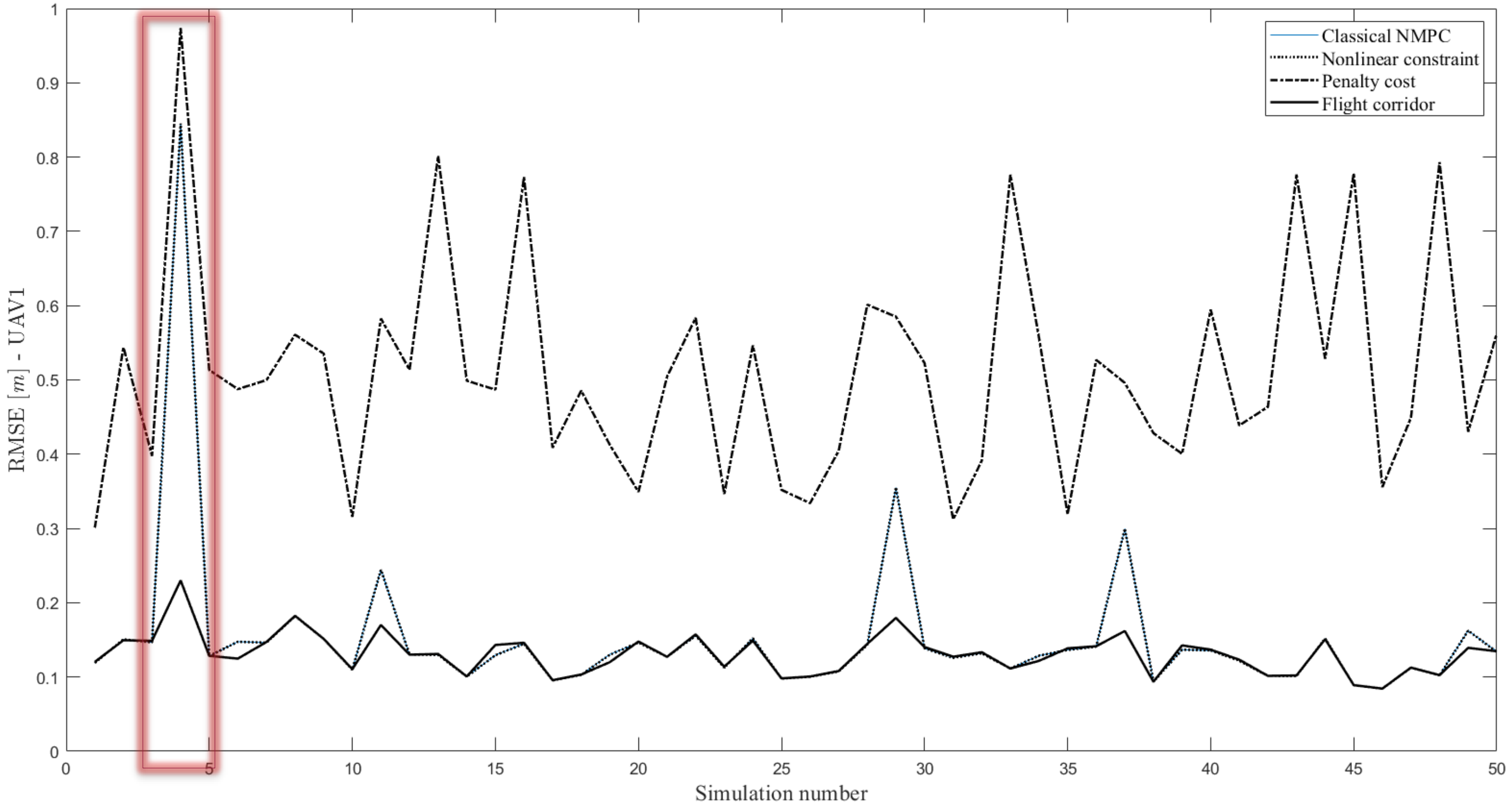






Monte Carlo simulations – 50 test cases

The worst case



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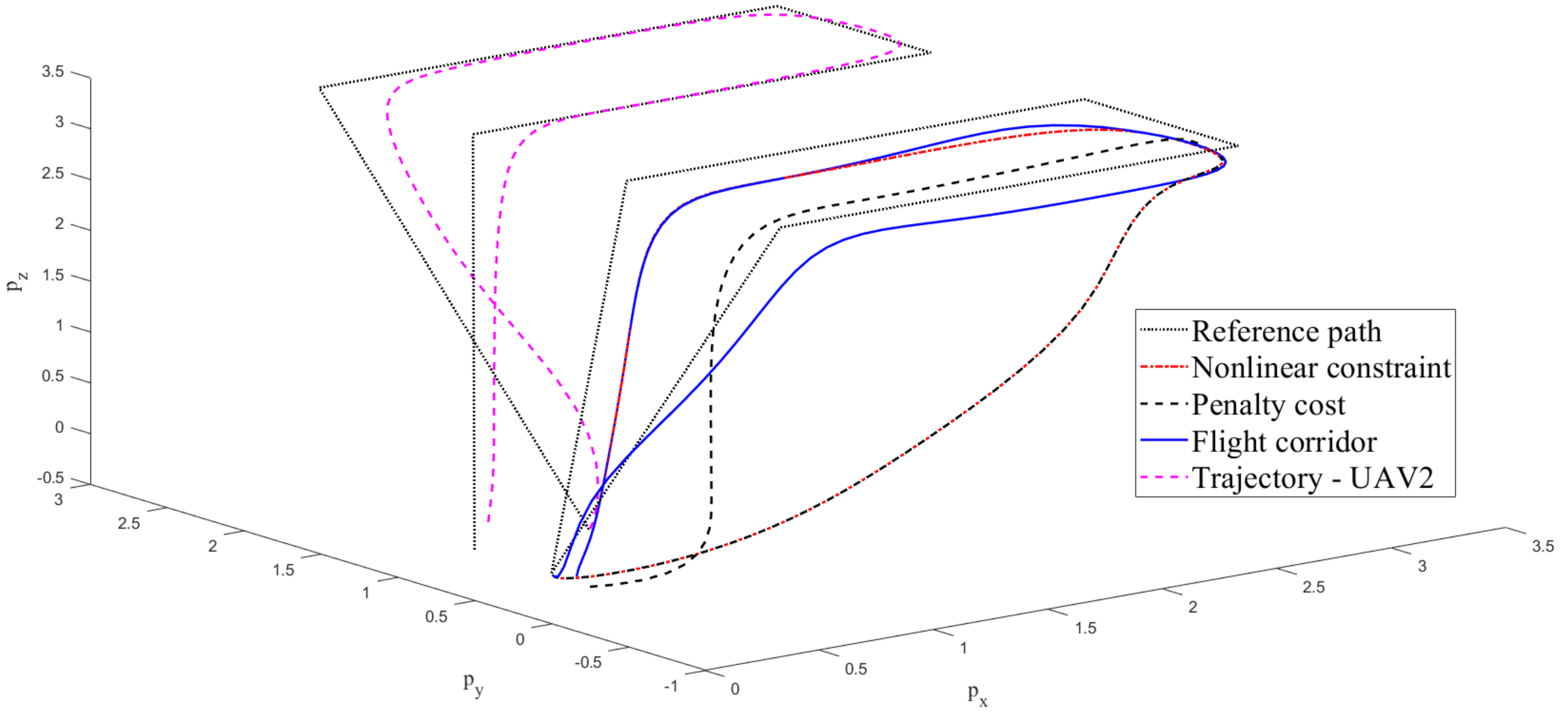
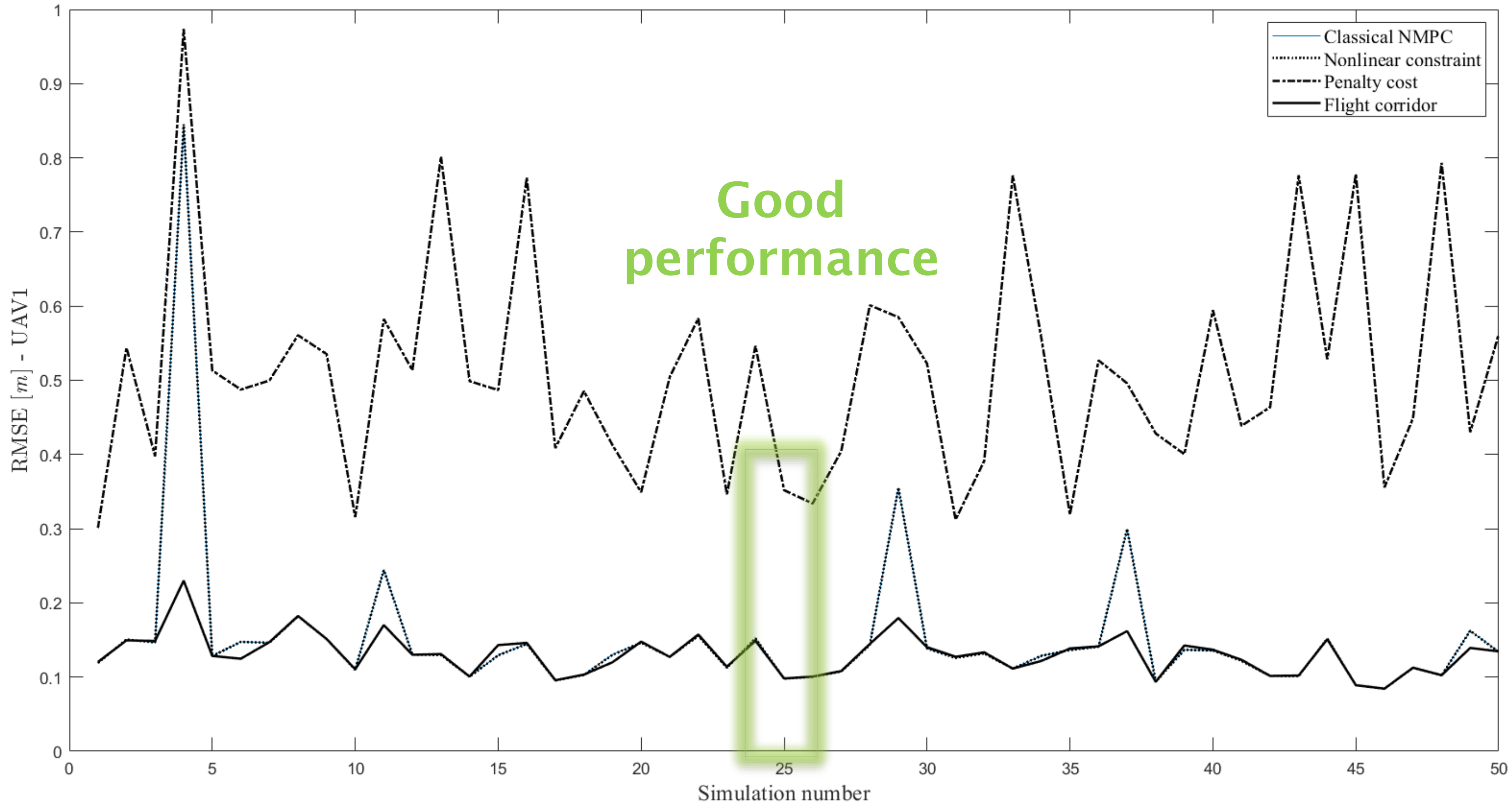
Monte Carlo simulations – 50 test cases

- The worst case – **test case 4**



Monte Carlo simulations – 50 test cases



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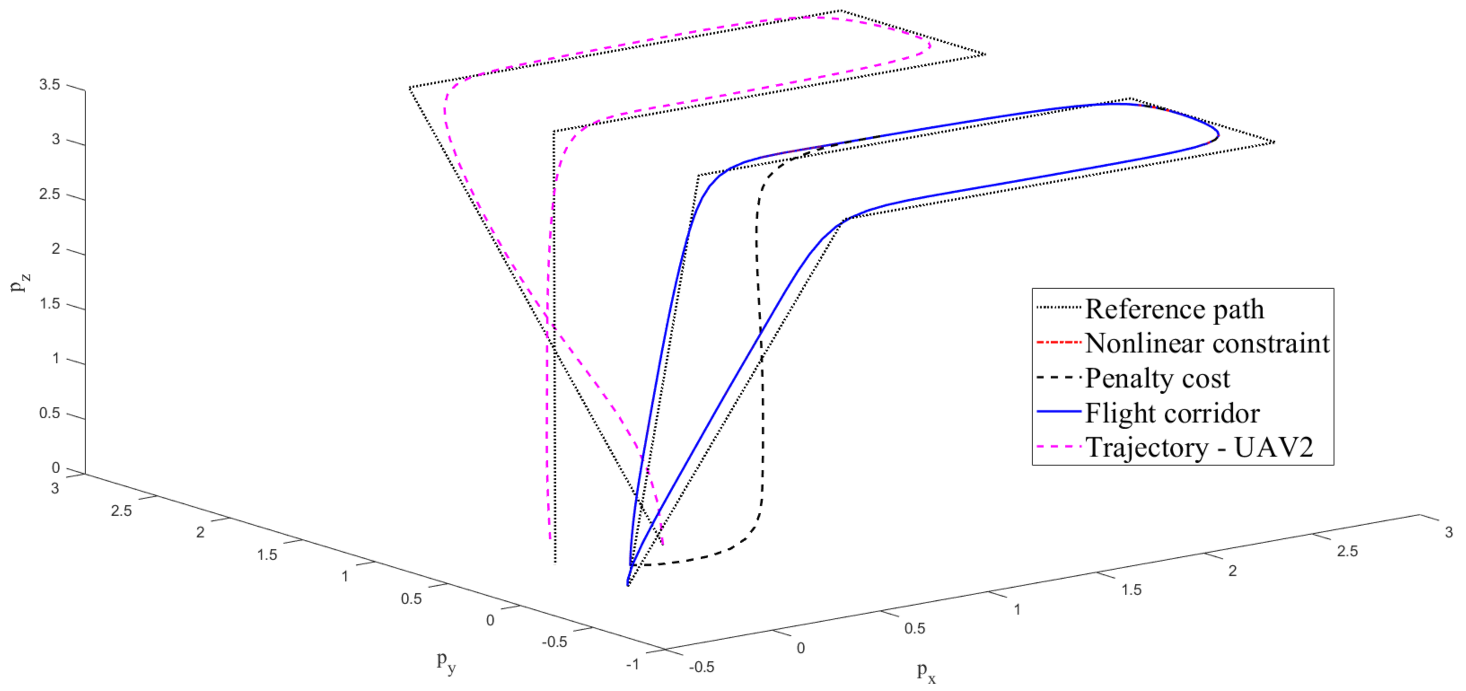
Monte Carlo simulations – 50 test cases

- Good tracking performance – **test case 25**



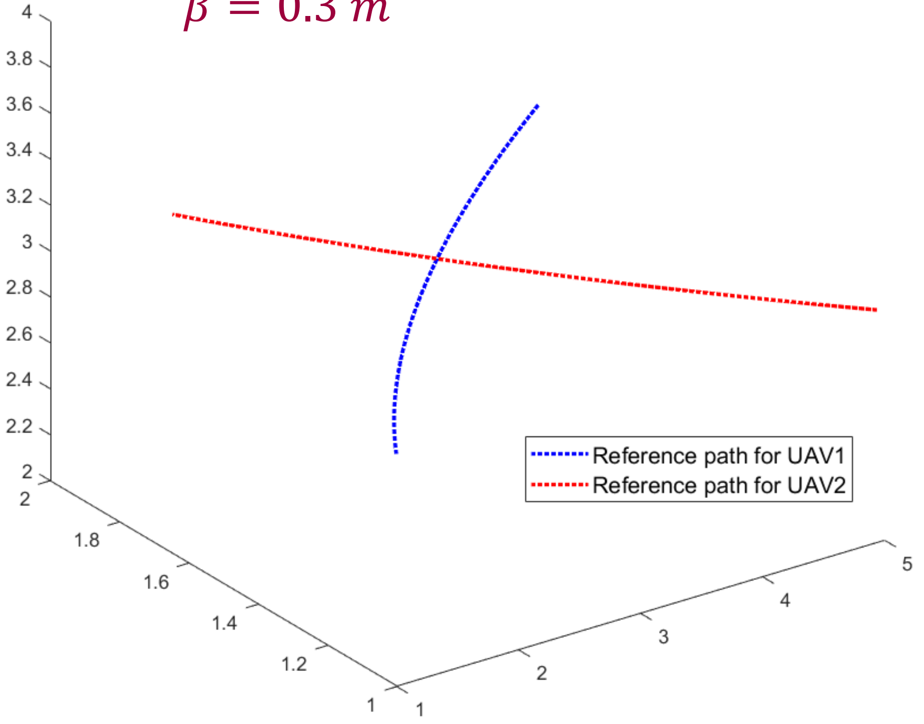
Results overview

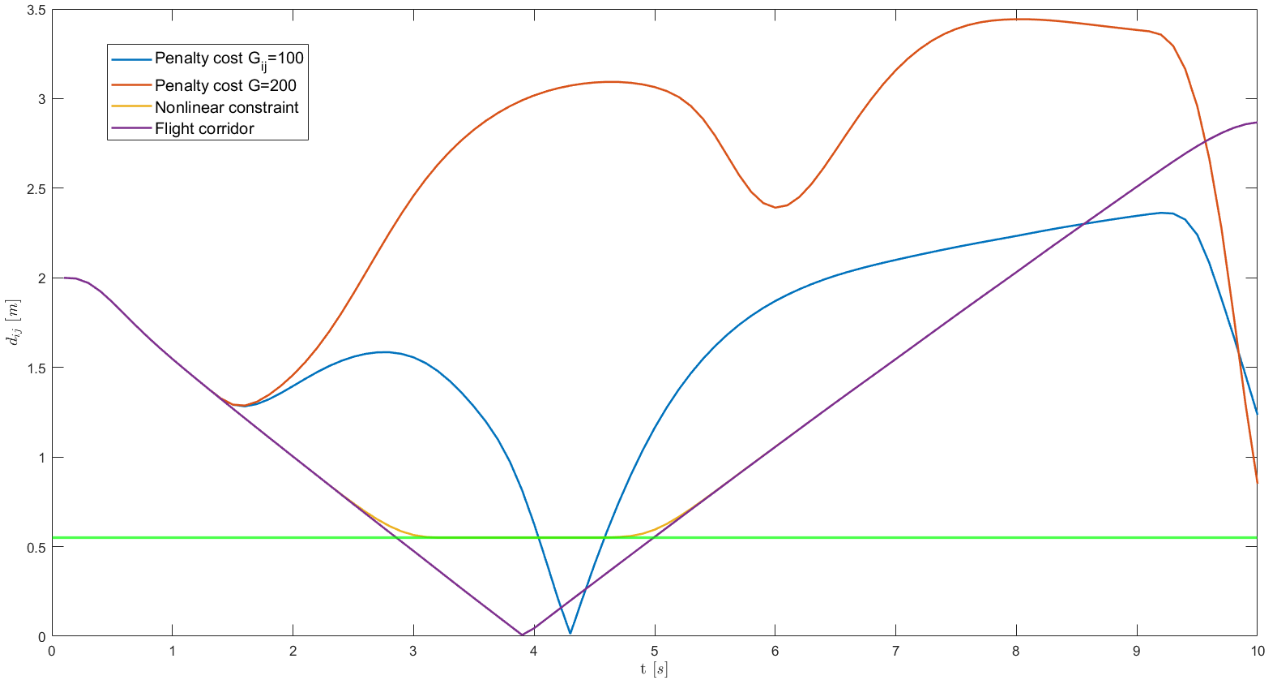
- Flight corridor results in the lowest RMSE
  - the highest precision, while compromising the safety distance
- Collision avoidance as a nonlinear constraint and as a penalty cost
  - Respecting the safety distance with increased CPU time

- **What if the planned paths intersect?**

Intersecting paths

$d_s = 0.55\ m$
$\beta = 0.3\ m$





- **Nonlinear constraint** – successful collision avoidance without further deviations

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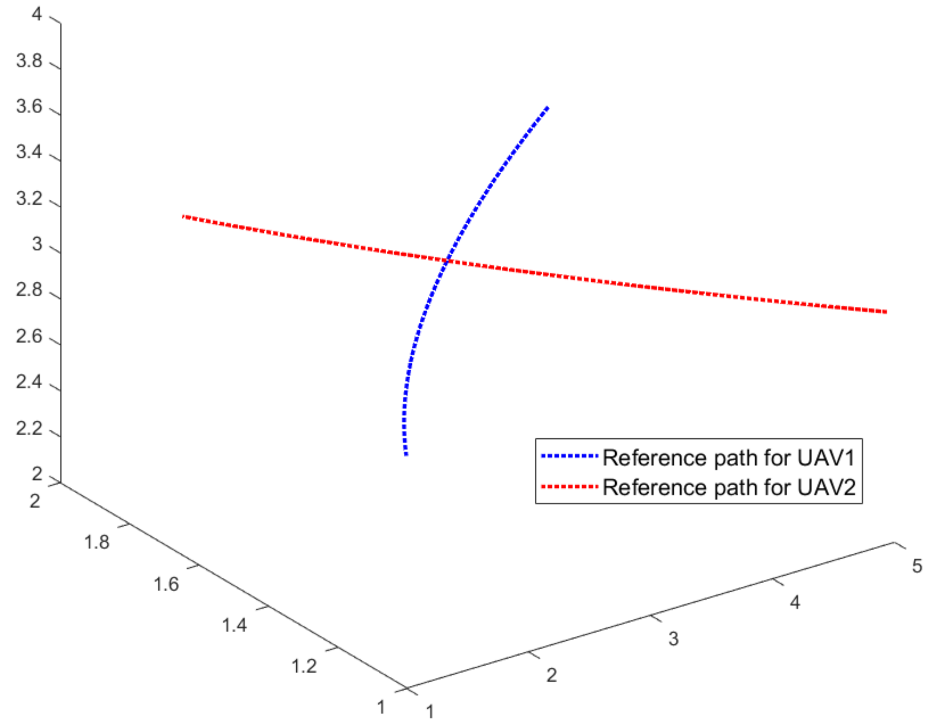
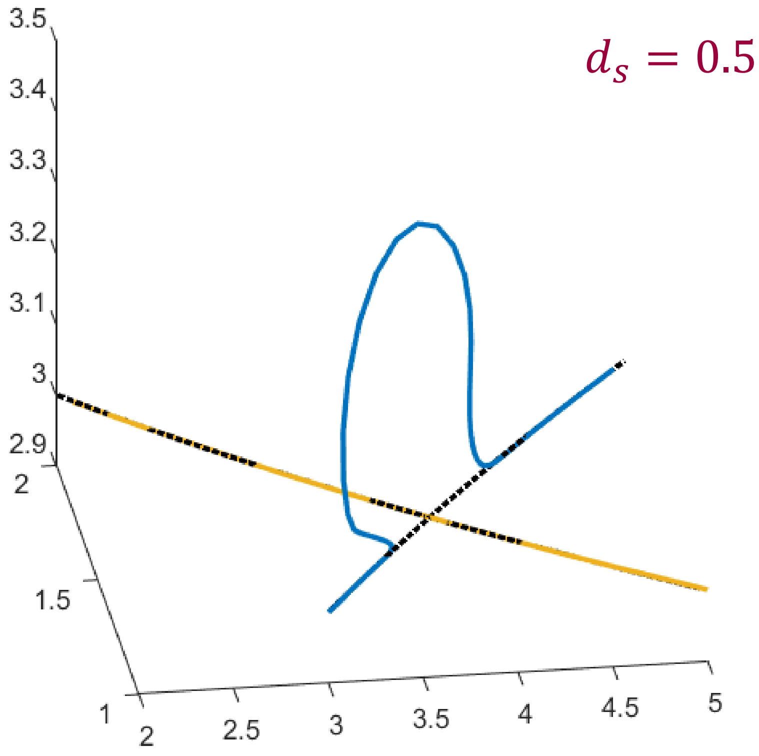
Intersecting paths





$d_s = 0.55\ m$

- **Nonlinear constraint** – successful collision avoidance without further deviations

6. Conclusion and perspectives

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Conclusion

- **Prioritized multi-UAV trajectory tracking with collision avoidance**
  - 3 distributed NMPC strategies:
    1. Collision avoidance as a nonlinear constraint
    2. Collision avoidance in the cost function
    3. Collision avoidance through a flight corridor
- The best trade-off between the performance and computational burden

  → **Flight corridor**

  - UAV remains inside the corridor despite the uncertainties and disturbances
  - Safety distance not respected → corridor design depends on the path configuration and UAV characteristics
  - Suitable for scenarios **without intersection** of the planned paths

Research perspectives

- **Mission planning:**
  - Planning from the battery perspective (ongoing work)
  - Online mission replanning
    - UAV failure
    - Insufficient battery level for mission completion
- **Trajectory tracking:**
  - Study of a multi-UAV mission
  - Online priority allocation
- Experimental validation

THANK YOU!